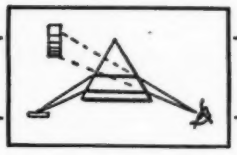
Journal of the

OPTICAL SOCIETY *of* AMERICA

VOLUME 33
No. 12



DECEMBER
1943

Design of a Telephoto-Magnifier . . . . . MAX REISS 641

Gaussian Optics and Gaussian Brackets . . . . . M. HERZBERGER 651

Compendium of Line Pairs and Operating Conditions Used in Spectrographic Analysis . . . . . EDWIN S. HODGE 656

Spectrographic Determination of Thorium in Tungsten Filament Wire . . . . S. L. PARSONS 659

Hydrogen Discharge Tube with Metal Capillary . . . . EGON LORENZ AND PAUL D. KUECK 663

Note on Ray Tracing by Machine . . . . . JAMES H. ELLIS 665

Sampling, Mixing, and Grinding Techniques in the Preparation of Samples for Quantitative Analysis by X-Ray Diffraction and Spectrographic Methods . . . . . JAMES W. BALLARD, H. I. OSHRY, AND H. H. SCHRENK 667

Graphical Representation of Small Color Differences . . . . . D. L. MACADAM 675

Letter to the Editor:

Note on Theoretical Color-Blindness Frequencies . . . . . JOHN R. PLATT 679

Proceedings of the Twenty-Eighth Annual Meeting of the Optical Society of America 680

Minutes of the Twenty-Second Meeting of the Board of Directors of the Optical Society of America, Incorporated 691

Author Index to Volume 33 692

Analytic Subject Index to Volume 33 694

Volume Table of Contents III

Published monthly
at Prince and Lemon Sts., Lancaster, Pa.
by the
AMERICAN INSTITUTE OF PHYSICS
Incorporated
175 Fifth Avenue, New York 10, N. Y.
FOR AND UNDER THE EDITORIAL SUPERVISION OF THE
OPTICAL SOCIETY OF AMERICA

THE OPTICAL SOCIETY OF AMERICA

Officers of the Society

A. H. PFUND, *President*
Johns Hopkins University
Baltimore, Maryland

GEORGE R. HARRISON, *Vice President*
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

ARTHUR C. HARDY, *Secretary*
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

W. W. GRAEPER, *Treasurer*
Bausch & Lomb Optical Company
Rochester 2, New York

The Board of Directors consists of the above officers; the Editor of the Journal; the Past President; and five directors-at-large.

A. G. WORTHING, *Past President*
University of Pittsburgh
Pittsburgh, Pennsylvania

GEORGE R. HARRISON, *Editor*
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Directors-at-Large

WALLACE R. BRODE
Ohio State University
Columbus, Ohio

SELIG HECHT
Columbia University
New York 27, New York

D. B. JUDD
National Bureau of Standards
Washington, D. C.

D. L. MACADAM
Eastman Kodak Company
Rochester 4, New York

R. A. SAWYER
Naval Proving Ground
Dahlgren, Virginia

Rochester, New York, Section

F. W. PAUL, *President*
P. W. STEVENS, *Vice President*
A. F. TURNER, *Secretary*
F. H. PERRIN, *Treasurer*

Councillors:

F. C. FAIRBANKS
J. H. MCLEOD
W. T. PERKINS
D. L. WILLIAMS
J. H. WEBB (*ex officio*)

Patrons

AMERICAN OPTICAL COMPANY
BAUSCH & LOMB OPTICAL COMPANY
EASTMAN KODAK COMPANY

Corporation Members

AMERICAN CYSTOSCOPE MAKERS, INC., New York, New York
AMERICAN OPTICAL COMPANY, Southbridge, Massachusetts
BAUSCH & LOMB OPTICAL COMPANY, Rochester, New York
BELL & HOWELL COMPANY, Chicago, Illinois
BELL TELEPHONE LABORATORIES, INC., New York, New York
BENDIX AVIATION CORPORATION, New York, New York
CENTURY LIGHTING, INC., New York, New York
CORNING GLASS WORKS, Corning, New York
E. I. DUPONT DE NEMOURS & COMPANY, INC., Chemical Department, Experimental Station, Wilmington, Delaware
EASTMAN KODAK COMPANY, Rochester, New York
FAIRCHILD AVIATION CORPORATION, Jamaica, New York
FISH-SCHURMAN CORPORATION, New York, New York
GAERTNER SCIENTIFIC CORPORATION, Chicago, Illinois
W. & L. E. GURLEY, Troy, New York
INTERCHEMICAL CORPORATION, New York, New York
KEUFFEL & ESSER COMPANY, Hoboken, New Jersey
NATIONAL CARBON COMPANY, Cleveland, Ohio
PERKIN-ELMER CORPORATION, Glenbrook, Connecticut
POLAROID CORPORATION, Cambridge, Massachusetts
PORTMAN MACHINE TOOL COMPANY, New Rochelle, New York
SIMPSON OPTICAL MANUFACTURING COMPANY, Chicago, Illinois
SPENCER LENS COMPANY, Buffalo, New York
ARTHUR H. THOMAS COMPANY, Philadelphia, Pennsylvania
ZENITH OPTICAL COMPANY, Huntington, West Virginia

THE OPTICAL SOCIETY OF AMERICA was organized in 1916 "*to increase and diffuse the knowledge of optics in all its branches, pure and applied, to promote the mutual interests of investigators of optical problems, of designers, manufacturers and users of optical instruments and apparatus of all kinds and to encourage cooperation among them.*" It aims to serve the interests of all who are engaged in any branch of optics from fundamental research to the manufacture of optical goods.

MEMBERSHIP

Regular Members: "Any person who has contributed materially to the advancement of optics is eligible for regular membership."

Associate Members: "Any person who is interested in optics is eligible for associate membership."

Corporate Members: "Any corporation desirous of encouraging the work of the Society is eligible for corporation membership."

Patrons: The payment, within any one year, of $1000 or more toward the expenses of the Society by any corporation or individual entitles such corporation or individual to the rank of patron.

The Society cordially invites to membership all who are interested in any branch of optics, either in research, in instruction, in optical or illuminating engineering, in the manufacture and distribution of optical goods of all kinds or in physiological and medical optics.

ANNUAL DUES

Regular members, $8.50; Associate members, $6.00; Corporate members, $50.00; Patrons, $1000.00. These dues include subscription to the Society's official journal, the *Journal of the Optical Society of America.*

CHANGES OF ADDRESS

All changes of address of members of the Optical Society of America should be communicated to the Secretary, Professor Arthur C. Hardy, Massachusetts Institute of Technology, Cambridge 39, Massachusetts.

Journal of the
OPTICAL SOCIETY
of AMERICA

VOLUME 33, NUMBER 12 DECEMBER, 1943

The Design of a Telephoto-Magnifier*

MAX REISS
Eastman Kodak Company, Rochester, New York

THIS paper contains a brief description of a number of interesting points that have arisen in the course of the design of a Galilean loupe of the Brücke type.

As is well known, straightforward algebraic solutions are only successful with simple systems of small aperture and field, and even in these cases final retouching by trigonometrical methods must be employed.

For more complicated systems the original set-up calculated by approximate algebraic methods invariably turns out to be unsatisfactory when checked by trigonometrical ray-tracing. The proper analysis of the contributions by the various surfaces to the final aberrations will indicate which changes in the construction of the system are likely to reduce the aberrations to tolerable amounts. At the present, due to the complexity of the systems involved and the great number of the variables, this seems to be the only appropriate procedure for attacking the problem.

While ordinary telephoto lenses are often encountered, telephoto lenses of small focal length and large back focal length (reversed telephoto lenses) are of more restricted use. They are useful for certain purposes, e.g., when mirrors or prisms or shutters require a considerable space between the last surface of the lens system and the focal plane. A further example is the tele-photo-magnifier (called Chevalier loupe[1] or more often Brücke loupe).

If we wish to observe an object with relaxed accommodation of the eye, we place the object at the focus of the lens and see the object under a magnification

$$m = 250/f'_{\text{mm}}$$

(250 mm being the conventional distance of distinct vision). Suppose we are given a single lens L_1 which we are using as a magnifier and we wish to increase the magnification without changing the object distance. If we add to the lens a telescopic system of angular magnification MA' the angle of emergence will be

$$U_2' = U_1' \cdot MA'.$$

This means that the whole system is now a magnifier having a magnification f_2'/f_3' times as great as that of L_1 alone, or

$$m = \frac{250}{f_1'} \cdot \frac{f_2'}{f_3'},$$

but the object distance is the same as before. (See Fig. 1.)

For telescopes to be used for a very small object distance when accommodation of the eye or focusing with the eyepiece is not sufficient, a

* Paper presented at the Twenty-Eighth Annual Meeting of the Optical Society of America, Pittsburgh, Pennsylvania, October 7–9, 1943.

[1] Ch. Chevalier, *Die Mikroskope und ihr Gebrauch*, edited by Fr. S. Kerstein (G. Basse, Quedlinburg and Leipzig, 1843).

BOSTON UNIVERSITY
COLLEGE OF LIBERAL ARTS
LIBRARY

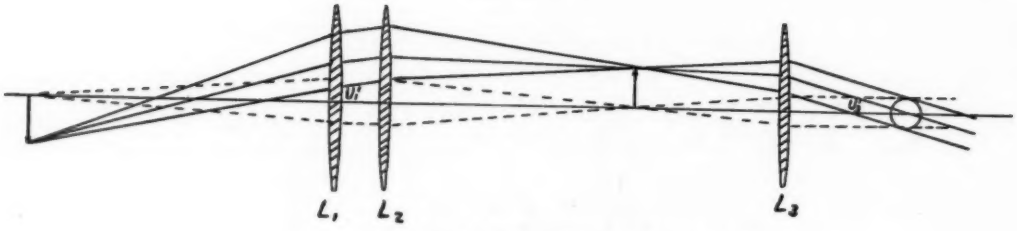


FIG. 1. Magnifier with positive eyepiece.

similar system has been suggested as far back as 1827 by C. A. Steinheil.[2] The light of a near object is rendered parallel by a special lens and then received by the telescope. This idea has then been used again by Henker and Rohr[3] for the design of a "telescope-loupe" which as a periscopic system has been manufactured by Zeiss.

We can, therefore, look upon this system as being either a magnifier of low magnification and, therefore, large object distance combined with a telescope, or as being a telescope used for a near object and combined with an extra lens which focuses the light at infinity. Or we can consider L_1 and L_2 as the objective and L_3 as the eyepiece of a compound microscope with a magnification corresponding to the total focal length

$$f' = \frac{f_1' f_2'}{\Delta}$$

(f_1', focal length of objective; f_2', focal length of eyepiece; Δ, distance of the back focus of the objective to the front focus of the eyepiece). This system, of course, gives an inverted image.

If we desire an erect image we would have to use a negative lens as an eyepiece. We can easily see that if we space the eyepiece so that the eye receives parallel light and if we keep f_1', f_2', and Δ constant, then because of $d = f_1' + f_2' + \Delta$ (f_2' is negative) the distance between the inner Gauss points of objective and eyepiece d is reduced by $2 \cdot f_2'$ as compared with the system with a positive eyepiece. This difference increases with the focal length of the eyepiece; i.e., with reduced magnification. For low magnifications, the use of a negative eyepiece therefore leads to a great reduction in the length of the system, which fact, together with the erect image constitutes a great advantage. (See Fig. 2.)

An additional advantage of using a negative lens is the possibility of satisfying the Petzval condition. For a system consisting of a positive and a negative lens which are separated by a fairly large distance, the Petzval condition $\sum(1/nf') = 0$ can be satisfied because, due to the separation of the lenses, the power contribution of even a strong negative lens is comparatively small. It is, therefore, possible to have a wider field of view without astigmatism than with a system using a positive eyepiece.

In spite of these advantages, microscopes with negative eyepieces are seldom used. One of the reasons is that they cannot be used with micrometer-scales due to the absence of a real image. But in some cases as in microscopes for low magnification used for anatomical specimens, the systems are still in use.

Brücke's original paper[4] published in 1851 only mentions that he used two achromats taken from the ocular of a microscope, while the eyepiece was that of an ordinary theater-glass. The object distance was 75 mm, the magnification 5, and the field of view was 14 mm. The system had a length of 90 mm and a diameter of 40 mm. As the chief advantage Brücke mentions the large object distance. He constructed the system in order to facilitate work on anatomical specimens and for the use of eye physicians in order to replace ordinary loupes, because with ordinary loupes the head of the observer casts a shadow on the patient's eye. Such loupes will be useful whenever a large object distance is necessary for the use of an illuminating lamp, tools, etc.

[2] C. H. Steinheil, Astronom. Nach. **5**, 359 (1827).
[3] O. Henker and M. von Rohr, Zeits. f. Instrumentenk. **29**, 280 (1909).
[4] E. Brücke, Wien. Sitz. **6**, 554 (1851). French translation: Ann. d. chim. et de phys. [3] **35**, 127 (1852).

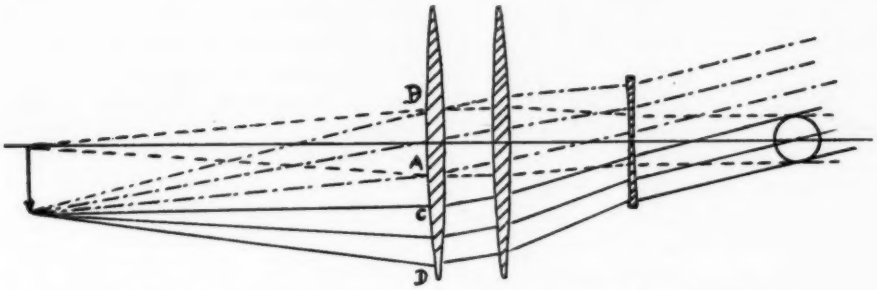


FIG. 2. Magnifier with *negative* eyepiece and therefore reduced over-all length.

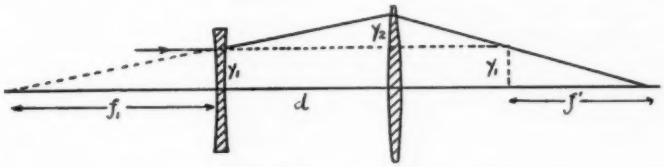


FIG. 3. Reversed telephoto lens (schematic).

The new design described below was, for various reasons, originally intended for use in monochromatic light. It was later achromatized for use with white light, as will be explained. (See Fig. 3.)

The three conditions for a prescribed focal length, back focal length, and Petzval sum are:

$$\phi=\phi_1+\phi_2-d\phi_1\phi_2,^{*}\quad s'=\frac{1}{\phi}(1-d\phi_1),^{**}$$

$$\frac{\phi_1}{n_1}+\frac{\phi_2}{n_2}=0.$$

This gives

$$\phi_2=\frac{-\phi}{(n_1/n_2)-s'\phi},\quad \phi_1=-\phi_2\frac{n_1}{n_2},\quad d=\frac{1}{\phi_1}(1-s'\phi).$$

For $s'=75$ mm, $f'=37$ mm, $n_1=1.61$, and $n_2=1.52$, this gives the thin-lens specification:

$$r_1=-r_2=-40.67,\quad r_3=-r_4=36.26,$$
$$d=34.24.$$

By assuming necessary thicknesses for each component, keeping the power of each component constant and spacing them so that the distance of the inner Gauss points remains constant we have:

$$r_1=-40.67$$
$$t_1=1.0\qquad n_1=1.61$$
$$r_2=+41.05$$
$$s=30.28\qquad\qquad f'=37.00.$$
$$r_3=36.26$$
$$t_2=10.00\qquad n_2=1.51$$
$$r_4=-32.88$$

As expected, this system has huge negative distortion and spherical aberration resulting from the large heights and consequently large angular deviations of the rays meeting the positive lens. This of course cannot be cured by changing the shape of the lenses. Replacing the single crown lens by two single lenses reduces both distortion and spherical aberration, but for the same reason as before, the undercorrection is still very large. We therefore had to divide the power of the positive lens still further, and for the next stage we tried a system having three positive lenses, one of them having a cemented overcorrecting surface (Fig. 4). The angular deviations at all the surfaces are now much smaller than before and this together with the overcorrecting action of the

---

* ϕ, total power of system; ϕ_1 and ϕ_2, power of lenses 1 and 2; d, separation of the lenses.

** The second equation follows from:

$$y_2/s'=y_1/f'\quad\text{or}\quad s'=(y_2/y_1)\cdot f'.$$

Furthermore:

$$y_2/(-f_1'+d)=-y_1/f_1'\quad\text{or}\quad y_2/y_1=(f_1'-d)/f_1';$$

therefore:

$$s'=f'[1-(d/f')].$$

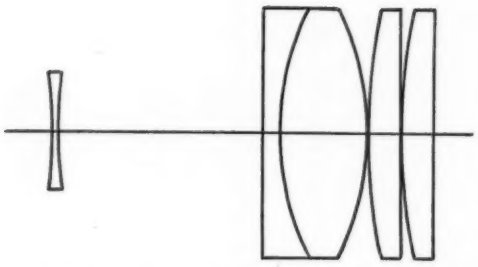


FIG. 4. Reversed telephoto lens with split back lens.

cemented surface accounts for the vast improvement in distortion and spherical aberration.

The specification of one of these forms is:

	t	
$r_1=-54.14$		
	0.96	
$r_2=54.14$		$n_1=n_3=1.61$
	33.11	
$r_3=\infty$		$n_4=n_6=n_5=1.51$
	3.00	
$r_4=36.82$		
	18.00	$f'=37.00$
$r_5=-39.53$		
	0.00	
$r_6=79.15$		
	4.8	
$r_7=\infty$		
	0.00	
$r_8=79.15$		
	6.00	
$r_9=\infty$		

Figure 5 shows the curves for the primary and secondary field as computed for a principal ray crossing the axis at a point 20 mm in front of the first surface. The field shows very large overcorrected astigmatism beyond 7° or 8°. In order to find the cause for this we computed the Seidel aberrations of the system. Table I gives the spherical contributions and astigmatic contributions of each surface to the final values for a nominal $f/4$ aperture and 10° half field. According to the relation*

$$AC'=SC'(i_{pr}/i)^2$$

a large contribution of AC' can be expected even at a surface where only a moderate amount of spherical aberration exists if $(i_{pr}/i)^2$ is large. This happens often in eyepieces where the incidence angles at the cemented surfaces of the eye lens are very large. The remedy consists in reducing SC' by choosing two glasses of small difference of index of refraction.[5] In this case surface 4 contributes a considerable amount of SC'. This being multiplied by $(i_{pr}/i)^2=2.908$ explains the large overcorrected astigmatism of the system. We, therefore, should expect a reduction of both spherical aberration and astigmatism by choosing as indices for the second and third lenses 1.541 and 1.51 (we had 1.61 and 1.51 before).

Figure 6 shows that the lens is nearly free from astigmatism and has a slightly inward curving field. In order to translate the curvature of field in the image space into diopters for the entering parallel light, we have to remember:

$$XX'=-f^2.$$

For a curvature of 1 mm this gives $X=1369$ mm or 0.73 diopters.

In order to secure sharply defined extra-axial

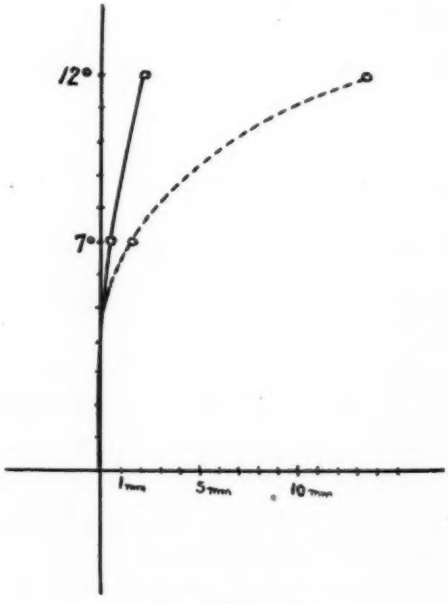


FIG. 5. Field curve.

* Using Conrady's notation, see reference 5, page 314.

[5] A. E. Conrady, *Applied Optics and Optical Design* (Oxford University Press, London, 1929), p. 504.

TABLE I.

Surface 1	2	3	4	5	6	7	8	9
$SC'=+0.00217$	$+0.0457$	$+0.0182$	$+0.0899$	-0.0267	-0.0136	-0.00413	$+0.000864$	-0.0320
$AC'=+0.00356$	$+0.128$	$+0.1071$	$+0.261$	-0.0442	-0.0704	-0.000437	$+0.00811$	-0.0305

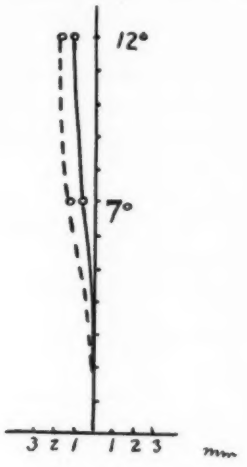


FIG. 6. Field curve.

image points the eye has to be accommodated for this distance. Generally this is only possible for light diverging towards the eye. That means that the field curvature, if any, computed for parallel light entering from the left has to be away from the lens (backwards). In case of a slightly inward curving field, a small shift of the object will compensate the field curvature at the expense of the sharpness of the points near the axis.

At this stage of the work it was decided to change the design so as to make usable for white light. It also seemed desirable to replace the two single lenses by a cemented doublet in order to reduce the number of glass-air surfaces. According to the rule that in a doublet of two thin lenses in order to free the system from axial and lateral color, both lenses have to be achromatized, the system was set up of three cemented doublets each of them achromatized,† (Fig. 7) although it was to be expected that the appreciable thickness of the positive elements and the steep angles might introduce large deviations from the rule mentioned above. For a thin achromatic lens, a change in object distance does not introduce any color residuals. But for thick lenses this is not fulfilled strictly and we, therefore, achromatized each element for parallel light, spacing the second lens so that parallel light emerges.** The system is, of course, free from axial color, but there is an extremely high residual of lateral color (0.60 mm). This could be expected by considering the fact that the primary contribution to the final lateral color and the contribution to the axial color are connected by:[6]

$$T_{ch}C' = L_{ch}C' u_k' (i_{pr}'/i'),$$

$$L_{ch}C' = y \cdot ni' \left(\frac{\delta n'}{n'} - \frac{\delta n}{n} \right) \Big/ n_k' u_k'^2$$

(k refers to the last surface).

This shows that (keeping the indices constant) $T_{ch}C'$ is proportional to y and i_{pr}'/i'. The principal ray meets the last cemented surface at an angle of about 60°. Reducing y and i_{pr}' will,

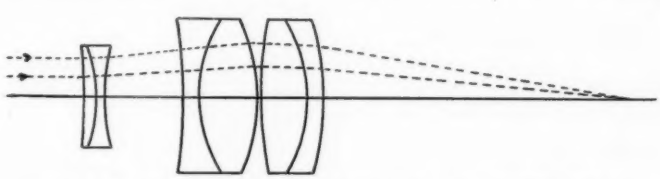


FIG. 7. Final set-up, achromatized for white light and containing 3 achromatic doublets.

† As regards the contribution of the negative lens to the lateral color we have to remember that for central passage of the principal ray, the contribution will be zero.

** The powers of the three doublets correspond roughly to those of the lenses in the last specifications, the two single lenses corresponding to the third cemented element in the present specification.

[6] See reference 5, p. 314.

FIG. 8. Field curve.

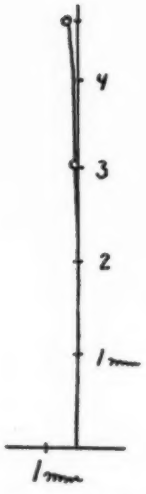


FIG. 9. Spherical aberration.

therefore, reduce the amount of lateral color. This was achieved by reducing s_1 so that all the rays meet the last two elements at considerably smaller heights. In reducing s_1 we reduce the power contribution of the positive elements. We therefore strengthened the second element until the old focal length was restored, all three elements being still achromats for parallel light. This immediately reduced the lateral color to +0.11 mm. The specification is now:

r	t	n	v
−60.59			
	2.00	1.617	36.6
−10.62			
	1.00	1.617	55.0
+44.58			
	13.65	1	
−69.70			
	3.00	1.649	33.8
+34.07			
	14.00	1.611	58.8
−27.84			
	0.00	1	
+70.00			
	14.00	1.611	58.8
−23.40			
	3.00	1.649	33.8
−88.52			

$f' = 41.1$

A moderate adjustment of the first and last cemented surface reduced the color-residuals to negligible quantities. The field shows hardly any astigmatism and only small inward curving up to 10° while from here on there is a small amount of astigmatism. The lens is spherically undercorrected and it does not yet fulfill the sine-condition.*

The Seidel analysis shows that surface 3 and 9 contribute most of the spherical aberration and coma. Bending of all 3 elements gives this set-up:

r	t	n	v
−111.07			
	2.00	1.617	36.6
−29.94			
	1.6	1.617	55.0
33.26			
	15.55	1	
−104.59			
	3.00	1.649	33.8
29.11			
	12.00	1.620	60.3
−31.13			
	0.00	1	
41.18			
	9.00	1.611	58.8
−68.50			
	3.00	1.649	33.8
−2646.0			

$f' = 41.1$

Back focal length: 62.4.

The curves show the field and spherical aberration (Figs. 8 and 9).

The curves for primary and secondary field alone are not sufficient for characterizing the quality of the extra-axial images at finite aperture because they only indicate the position of the tangential and sagittal focal lines for small apertures. Even for small apertures the astigmatic curves lose much of importance due to the increasing depth of focus. For a full charac-

* The lens, being of the reversed telephoto-type, has its nodal points not even approximately coinciding with the entrance and exit pupil and we therefore have to use the revised sine-condition by Staeble and Lihotzky:

$$\Delta F - \Delta S \frac{F}{S' - S_{pr}'} = 0;$$

ΔS = spherical aberration, ΔF = difference of focal lengths $Y/\sin U' - (Y/U')$, S' = intersection length of marginal ray, S_{pr}' = distance of exit pupil from the last surface.

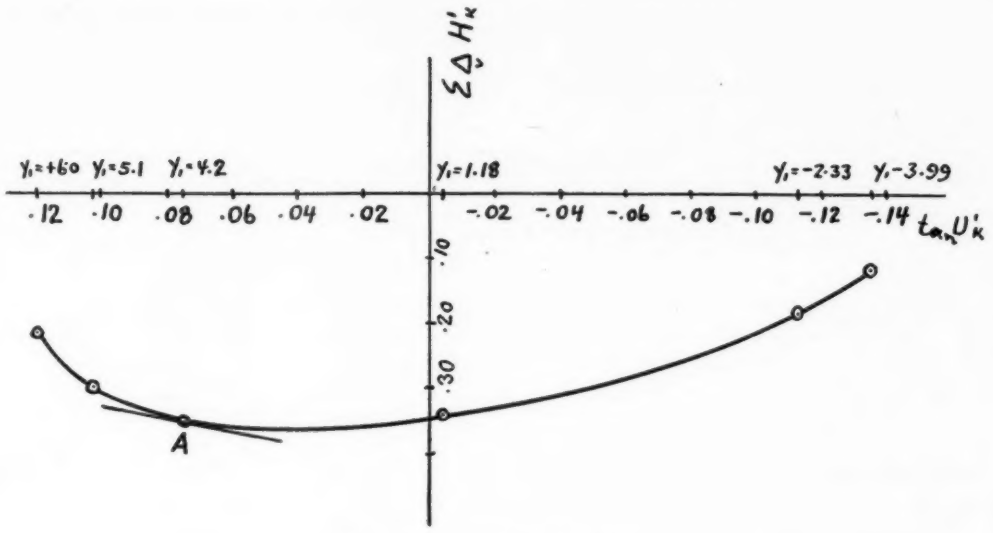


FIG. 10. Intersection-height curve.

terization of the performance of the lens the intersections of individual rays with the focal plane have to be computed.

In our present case Fig. 10 represents the intersection height in the focal plane as a function of the angle of emergence U_k' for a bundle of rays entering at 12°. As is well known the slope of the tangent at any point of this curve is equal to the distance from the focal plane to the primary focus as computed along that chosen ray. It is therefore clear that a computed field curve only refers to a small bundle of rays along a particular ray and a so-called "flat field" only means that the tangent is parallel to the abscissa for a single point of the intersection-height curve.

The field curves given in Fig. 8 correspond to the tangent at the point of A of Fig. 10, the slope being almost zero, but the curve as a whole clearly shows the large amount of coma still present in the system. The fulfillment of the sine condition guarantees freedom from coma only for small angles.

For a "perfect" intersection-height curve all points should lie on a straight line parallel to the abscissa at a height corresponding to the ideal height f' tan U_1. Any departure from that curve can in the general case be looked upon as a combination of curvature of field, coma, and spherical aberration of the oblique pencil. The difference between the actual intersection height of any ray and the ideal height can be called distortion of that ray. In order to correct the shape of the intersection-height curve we can compute the Seidel contributions to the distortion of a number of rays due to each surface and find out which surfaces contribute essentially to the distortion of the rays.* As in all similar cases where Seidel aberrations are used in order to judge the aberrations at higher angles and higher apertures this method is only of restricted value. An alternative method[7] which has been used in our case consists in using the data of the trigonometrically traced rays for computing the contribution of each surface to the final distortion.

In the following we shall call the distortion $\Delta H'$ of a ray the difference between the actual intersection height in the focal plane and the height according to paraxial magnification m (Lagrange height). See Fig. 11.

$$\Delta H' = (l' - s') \tan U_k' - H_1 m.*$$

For an object at infinity

$$\Delta H' = (l' - s') \tan U_k' + f' \tan U_1.$$

---

[7] M. Berek, *Grundlagen der praktischen Optik* (Walter de Gruyther and Company, 1930).

* The l' is the image distance of the principal ray from the respective surface.

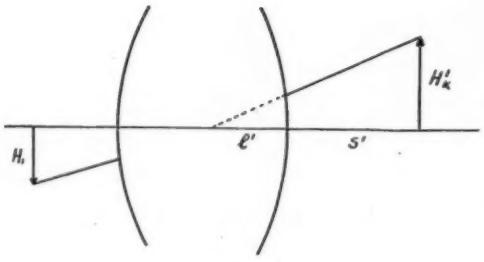


FIG. 11.

We now wish to compute the contribution of each surface to the final distortion. We assume the object to be at infinity.

FIRST SURFACE

After refraction at the first surface the distortion is

$$\Delta H_1' = (l_1' - s_1') \tan U_1' + f_1' \tan U_1$$

(f_1' is the focal length of the first surface). If the following surfaces did not introduce any distortion but only magnified according to paraxial magnification then the contribution of the first surface to the final distortion would be

$$\underset{1}{\Delta H_k'} = m_2 m_3 \cdots m_k [(l_1' - s_1') \tan U_1' + f_1' \tan U_1)]$$

or

$$\underset{1}{\Delta H_k'} = \prod_2^k m_v \cdot (l_1' - s_1') \tan U_1' + \prod_2^k m_v \cdot f_1' \tan U_1.$$

SECOND SURFACE

We now assume that the second surface also introduces distortion. Then the distortion in the image after the second refraction is:

$$\Delta H_2' = (l_2' - s_2') \tan U_2' + f_2' \cdot \tan U_1$$

(f_2' is the focal length of the first two surfaces). In order to get the distortion after the second surface which is due to the *second surface alone* we have to subtract the distortion after the first surface as magnified by the second surface. Or: Contribution of the second surface to the distortion after the second surface: $(l_2' - s_2') \tan U_2' + f_2' \tan U_1 - m_2[(l_1' - s_1') \tan U_1' + f_1' \tan U_1]$. If we again assume that the following surfaces do not introduce any distortion but only magnify the contribution of the second surface, then we get:

Contribution of the second surface to the final distortion:

$$\underset{2}{\Delta H_k'} = \prod_3^k m_v \cdot [(l_2' - s_2') \tan U_2'] + \prod_3^k m_v f_2' \tan U_1 - m_2 \prod_3^k m_v \cdot \underset{1}{\Delta H_1'}.$$

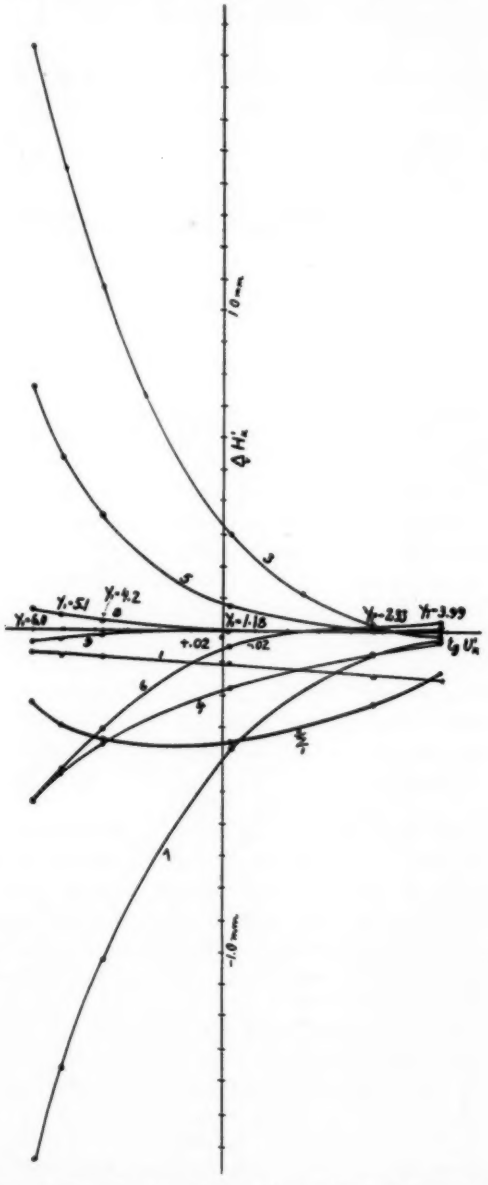


FIG. 12. Contribution of each surface to the final distortion.

The last term is:

$$m_2 m_3 \cdots m_k \Delta H_1' = \underset{1}{\Delta H_k'}.$$

Therefore:

$$\underset{2}{\Delta H_k'} = \prod_3^k m_v (l_2' - s_2') \tan U_2' + \prod_3^k m_v \cdot f_2' \tan U_1 - \underset{1}{\Delta H_k'}.$$

We have to remember:

$$f' = \frac{n_1}{n_k'} \left(s_1' \prod_{v=2}^{k} \frac{s_v'}{s_v} \right)_{s_1 = \infty}.$$

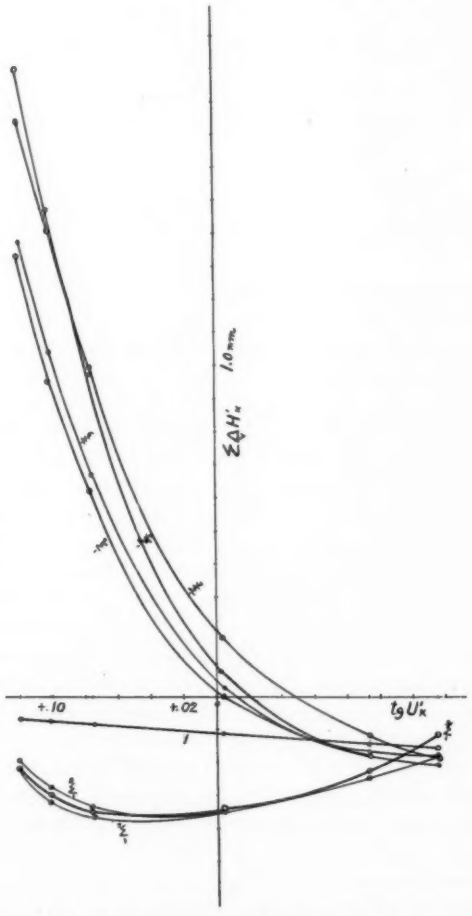


FIG. 13. Sum of the contribution of each surface to the final distortion for successive surfaces.

And the magnification:

$$\prod_{v=1}^{k} m_v = m_1{}^k = \frac{n_1}{n_k'} \prod_{v=1}^{k} \frac{s_v'}{s_v}.$$

Therefore:

$$\prod_3^k m_v = \frac{n_3}{n_k'} \prod_{v=3}^{k} \frac{s_v'}{s_v} = \frac{n_2'}{n_k'} \cdot \frac{s_2}{s_2'} \prod_{v=2}^{k} \frac{s_v'}{s_v} = \left(\frac{n_1}{n_k'} s_1' \prod_{v=2}^{k} \frac{s_v'}{s_v} \right) \left(\frac{n_2'}{n_1} \frac{s_2}{s_1' s_2'} \right).$$

Now:

$$\frac{n_1}{n_2'} s_1' \frac{s_2'}{s_2} = f_2' \quad \text{and} \quad \frac{n_1}{n_k'} \cdot s_1' \prod_{v=2}^{k} \frac{s_v'}{s_v} = f'.$$

Therefore:

$$\prod_3^k m_v = f'/f_2'$$

and

$$\underset{2}{\Delta H_k'} = \frac{f'}{f_2'} (l_2' - s_2') \tan U_2' + f' \tan U_1 - \underset{1}{\Delta H_k'}.$$

Generally:

$$\underset{v}{\Delta H_k'} = \frac{f'}{f_v'} (l_v' - s_v') \tan U_v' + f' \tan U_1 - \sum_{u=1}^{v-1} \underset{u}{\Delta H_k'}.$$

The last term represents the sum of the contributions to the final distortion of all the surfaces preceding the vth surface.

The U values are given by the trigonometrical tracing of the ray, the f_v' and s_v by the tracing of the paraxial ray.

The next diagram (Fig. 12) shows the contribution $\underset{v}{\Delta H_k'}$ of all the 9 surfaces to the final distortion as a function of the angle of emergence U_k'. The heights on the first surface (Y_1) are also given. A constant abscissa corresponds to a certain ray and all the ordinates for that constant abscissa represent the distortion contributions of the various surfaces. If the contributions of all the surfaces were linear functions of U_k' they would of course add up to a linear function of U_k'. This means that even in the case of high aberrations at each surface the emerging bundle of rays would be free from coma. But generally the contributions are not linear and are not likely to add up to a straight line but to a curve

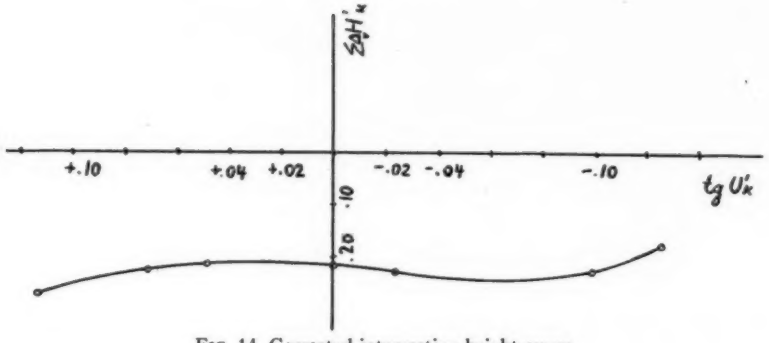


FIG. 14. Corrected intersection-height curve.

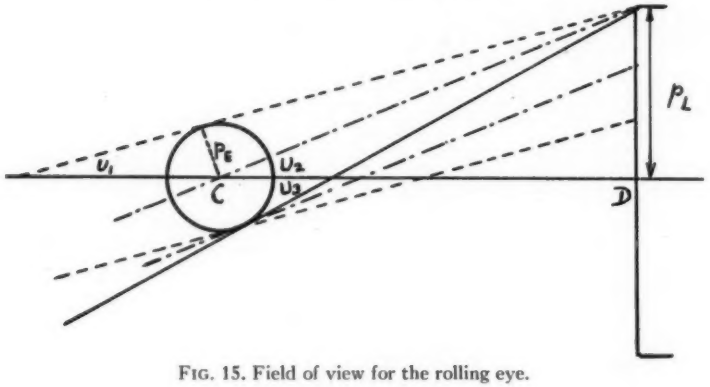


FIG. 15. Field of view for the rolling eye.

indicating coma. It is very instructive to add up the aberrations successively, and plot them as, 1, $\sum$1–2, $\sum$1–3, $\sum$1–9 against U_k' resp. (Fig. 13). For very large negative y values (lower rays) all the contributions are small (angles of incidence small) but for more positive values surfaces 4, 6, and 7 contribute a large amount of undercorrection, surfaces 3 and 5 large overcorrection. The figure for $\sum_{9}\Delta H_k'$ shows the very large amount of overcorrection still present after the rays have passed the 6th surface. This overcorrection has to be compensated by the undercorrection of r_7 and we therefore have to expect a large zone. This is exactly what happens In order to flatten this curve we shall therefore have to reduce the action of the overcorrecting *and* undercorrecting surfaces simultaneously. Weakening of surfaces 3, 5, 6, and 7 immediately improves the lens (Fig. 14). This of course requires a careful balancing of the changes in order to hold the other aberrations within tolerable limits.

A point of great importance for the design of this system is this:

Let us choose a certain diameter for the positive lenses so that the emergent parallel pencil is wide enough to fill the pupil of the eye. Then the virtual exit pupil restricts the field of view drastically, a fact well known from the Galilean telescope. In order to increase the field of view the diameter of the positive lenses has to be increased (Fig. 2). The lower pencils thus admitted can now enter the eye but these pencils, having large angular deviations are subject to large aberrations. Furthermore, the design of the system could be much facilitated if we could assume that the position of the eye will be constant. But this is not the case and it has to be assumed that the eye will travel in the direction of the axis and perpendicular to it. We therefore have to correct a much wider pencil than the pencil $C-D$ (Fig. 2). This is the reason why we had to correct the intersection-height curve so that it should be flat over a considerable range of U_k' values.

If we assume that the size of the positive lens is large enough to admit a pencil which fills the whole negative lens then we can consider the negative lens as being the exit-pupil of the system excluding the pupil of the eye. The field of view will be determined by the relative size and position of the pupil of the eye and the exit-pupil (Fig. 15).

Suppose we move the eye to a certain position close to the last surface of the loupe (parallel light emerging) and let us further assume that the radius of that surface (exit-pupil) be larger than the pupil of the eye. Now suppose we wish to see the outer part of the field without moving the head, then the eye rolls in its socket around the center of rotation C so that the axis of the eye will be parallel to the direction of the parallel bundle of rays emerging from the lens. In order to get an idea of the light-intensity corresponding to various obliquities we draw a circle around C with a radius equal to the radius of the eye-pupil (p_E).

1. Full illumination: From obliquity 0 up to obliquity u_1 the full eye-pupil will be filled with light. u_1 is given by drawing the tangent from the edge E of the lens to the circle.

The equation to this tangent gives for u_1:

$$\tan u_1 = p_L d - p_E(p_L^2+d^2-p_E^2)^{\frac{1}{2}}/(d^2-p_E^2).*$$

2. Principal ray still admitted: If we stipulate that the principal ray (ray through C) has to be admitted then a pencil half as wide as before will be admitted. We get the corresponding angle by joining C and E. The angle $\tan u_2 = p_L/d$.

3. The limiting angle at which any light at all arrives is given by drawing the lower tangent from E to the circle.

$$\tan u_3 = p_L d + p_E(p_L^2+d^2-p^2{}_E)^{\frac{1}{2}}/(d^2-p_E^2).$$

This means that by rolling the eye we have a region of constant aperture from 0 to u_1. From u_1 on the aperture decreases up to u_2. From u_2 to u_3 the principal ray is not admitted any more, the pencil pierces the outer part of the pupil and it is probable that these rays will not form a good image. In this case a transversal shift of the eye will have to compensate for that. For $p_L = 4$ mm, $p_E = 2$ mm, $d = 18$ mm, we get:

$$u_1 \quad 6.4°, \quad u_2 \quad 12.5°, \quad u_3 \quad 18.7°.$$

· · · In conclusion I should like to thank Professor R. Kingslake for many valuable discussions and suggestions in the course of this work.

* The distance $C-D=d$. The equation to the tangents from a point (x_1y_1) to the circle $x^2+y^2=r^2$ is

$$y-y_1 = \frac{-x_1y_1 \pm r(x_1^2+y_1^2-r^2)^{\frac{1}{2}}}{r^2-x_1^2}(x-x_1).$$

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 33, NUMBER 12 DECEMBER, 1943

Gaussian Optics and Gaussian Brackets*†

M. HERZBERGER
Kodak Research Laboratories, Eastman Kodak Company, Rochester, New York

Gauss invented an algorithm for certain arithmetical purposes, which has various applications in optical problems. For instance, it is sometimes of value to see how the focal length, magnification, and back focus of a lens system vary with the change of one of the construction elements (thickness, radius, or refractive index). It is sometimes very useful to have explicit expressions for these significant lens characteristics as functions of the construction elements. However, the usefulness of these Gaussian brackets is not limited to Gaussian optics. The image errors for finite rays in the new form recommended by the author can be expressed with their help.

IN the nomenclature of geometrical optics the term "Gaussian optics" is often used to include all problems in which only first-order effects are considered, i.e., where all monochromatic aberrations are neglected and all apertures and angles are vanishingly small. Thus we honor the name of the great mathematician C. F. Gauss (1777–1855), who, with several of his

* Paper presented at the Twenty-Eighth Annual Meeting of the Optical Society of America, Pittsburgh, Pennsylvania, October 7–9, 1943.
† Communication No. 948 from the Kodak Research Laboratories.

contemporaries, extended "thin-lens optics" to systems with finite thicknesses and separations.

By a coincidence, in modern parlance the name of Gauss is also given to a set of recursion formulae developed by L. Euler[1] and republished by Gauss.[2] Euler had developed these formulae, among other reasons, to solve the problem of finding the common denominator of two numbers. It appears to have escaped Gauss's attention, however, that the so-called "Gaussian brackets," originally a purely mathematical concept, can be highly useful in handling problems of Gaussian optics, so useful indeed as to merit the attention of present-day optical designers as a time-saving device in computing the effects of changes in surface power, thickness, and separation on focal length, magnification, and object and image distances. On the other hand, the statement of paraxial optical problems in the form proposed for the Gaussian bracket method is a good point of departure for a broader understanding of the image errors of a general optical system.[3-5] For simplicity, the following discussion is confined to rays lying in the meridian plane.[5]

This statement is as follows:

Let x = distance (paraxial) between intersections of axis and entering ray with plane of first vertex normal to axis;
ξ = inclination angle of entering ray;
x' = distance between intersection of axis and emerging ray with plane of last vertex;
ξ' = inclination angle of emerging ray.

Then

$$x' = ax + b\xi, \qquad \xi' = cx + d\xi, \tag{1}$$

where a, b, c, and d are constants such that

$$ad - bc = 1. \tag{2}$$

(Cf. Eqs. (1) and (8) of reference 1.)

[1] L. Euler, *Institutiones Calculi Differentialis cum eius usu in Analysi Finitonum de Doctrina Serierum* (Imperial Academy of Science, Petrograd, 1755).
[2] C. F. Gauss, *Disquistiones Arithmeticae* (1801).
[3] M. Herzberger, Am. Math. Soc. **53**, 218 (1942).
[4] M. Herzberger, Quart. App. Math. **1**, 1 (1943).
[5] The methods developed here were first made public in a comment by the present author on a paper by S. Rosin and O. H. Clark [J. Opt. Soc. Am. **31**, 198–201 (March, 1941)] presented at a meeting of this Society. In this paper, Rosin and Clark had given the data of Gaussian optics in explicit form, with the help of determinants.

GAUSSIAN OPTICS

These four constants, a, b, c, and d, will be expressed by Gaussian brackets, but first the usual data of Gaussian optics will be extracted from Eqs. (1) and (2).

The focal length is found by setting $\xi = 0$ in (1), giving

$$x/\xi' = 1/c, \tag{3}$$

or $c = \xi'/x$ is the power of the system. The back focus, or distance of the focal point from the last lens surface, is given by

$$s'_F = x'/\xi' = a/c. \tag{4a}$$

Similarly, the front focus is found by inserting $\xi' = 0$ into (1):

$$s_F = x/\xi = -(d/c). \tag{4b}$$

Now, let the ray come from an object distance s. Then

$$s = x/\xi, \quad s' = x'/\xi', \tag{5}$$

or

$$s' = (as + b)/(cs + d). \tag{6}$$

The magnification m is given by

$$m = \xi/\xi' = 1/(cs + d) = a - cs'. \tag{7}$$

On the other hand, if m is given, s and s' are obtained from

$$s = (1/c)[(1/m) - d], \qquad s' = (1/c)[a - m]. \tag{8}$$

Equation (8) contains (4) for the special cases where $m = 0$ and $1/m = 0$, respectively.

The constants a, b, c, and d contain implicitly the constants of the system. Each constant is a function of the individual surface powers, thicknesses, and separations in the system. To give them explicitly, Eq. (1) may be decomposed into a series of refraction and transposition equations, one for each surface and each separation (or thickness), respectively.

Let

$$\phi_k = (n_{k,k+1} - n_{k-1,k})/r_k = \text{the power of the } k\text{th surface}, \tag{9}$$

and

$d_{k,k+1}$ = distance between the kth and the $(k+1)$st surface

(where $n_{01}=1$, and $\xi_{k,k+1}$ represents the product of $n_{k,k+1}$ and the inclination angle).

Then

$$\begin{aligned} \xi_{12}&=\xi_{01}+\phi_1 x_1, && \text{(first refraction)}\\ x_2&=x_1-(d_{12}/n_{12})\xi_{12}, && \text{(first transposition)}\\ \xi_{23}&=\xi_{12}+\phi_2 x_2, && \text{(second refraction)} \end{aligned} \tag{10}$$

and so forth.

GAUSSIAN BRACKETS

If we have k numbers $a_1, \cdots a_k$, the Gaussian brackets are defined by the recursion formula:

$$[a_1, \cdots a_k]=a_1[a_2, \cdots a_k]+[a_3, \cdots a_k]. \tag{11}$$

They are linear functions of each of the elements, formed as follows:

The first term is the product of all k members; after it come all possible products of $(k-2)$ members in which the numbers have alternately odd and even indices in ascending order, each starting with an odd index; then all possible products of $(k-4)$ members likewise have successively higher alternating odd and even indices, each starting with an odd index; and so on. If the bracket has an odd number of members, it ends with the sum of all members of odd index; if it has an even number, it ends with unity.

Thus:

$$\begin{aligned} &[\ ]=1, \quad [a_1]=a_1, \quad [a_1, a_2]=a_1a_2+1,\\ &[a_1, a_2, a_3]=a_1a_2a_3+a_1+a_3,\\ &[a_1, a_2, a_3, a_4]=a_1a_2a_3a_4+a_1a_2\\ &\qquad\qquad +a_1a_4+a_3a_4+1,\\ &[a_1, a_2, a_3, a_4, a_5]=a_1a_2a_3a_4a_5+a_1a_2a_3\\ &\qquad +a_1a_2a_5+a_1a_4a_5+a_3a_4a_5+a_1+a_3+a_5. \end{aligned} \tag{12}$$

There are several interesting properties of these brackets. The proof of each of these laws follows the same pattern. The law is assumed to be correct for a certain value of n and proved to be correct for $n+1$ (principle of induction), using the defining Eq. (11). The reader may fill in the necessary steps.

I. The Gaussian bracket can be developed linearly in any of its members. That is,

$$\begin{aligned} &[a_1, \cdots a_k]=C_1a_\lambda+C_2,\\ &C_1=[a_1, \cdots a_{\lambda-1}][a_{\lambda+1}, \cdots a_k],\\ &C_2=[a_1, \cdots (a_{\lambda-1}+a_{\lambda+1}), \cdots a_k]. \end{aligned} \tag{13}$$

C_1, the coefficient of change of the bracket with respect to member a_λ, is the product of the bracket up to a_λ by the bracket containing the members beyond a_λ. C_2 represents the form to which the original bracket degenerates when one member becomes zero. It is a bracket of $(k-2)$ members in which the two members adjacent to the vanished bracket have been combined by addition into a single member. When $\lambda=1$, Eq. (13) reduces to the defining Eq. (11), since $(a_3\cdots a_k)$ is what is left when the first two members are omitted from the original bracket. When $\lambda=k$, Eq. (13) reduces to

$$[a_1, \cdots a_k]=a_k[a_1, \cdots a_{k-1}]+[a_1, \cdots a_{k-2}]. \tag{14}$$

II. The Gaussian bracket is symmetric in that

$$[a_1, \cdots a_k]=[a_k, \cdots a_1]. \tag{15}$$

III. It obeys the determinant formula:

$$[a_1, \cdots a_k][a_2, \cdots a_{k-1}] -[a_1, \cdots a_{k-1}][a_2, \cdots a_k]=1. \tag{16}$$

APPLICATIONS

Equations (10) express in three steps the course of a meridional ray through an optical system consisting of two surfaces, e.g., a single thick lens. Elimination of the intermediate ray data leads to

$$\begin{aligned} x_2&=x_1\{1-(d_{12}/n_{12})\phi_1\}-(d_{12}/n_{12})\xi_{01},\\ \xi_2'&=x_1\{\phi_1+\phi_2-(d_{12}/n_{12})\phi_1\phi_2\}\\ &\qquad +\{1-(d_{12}/n_{12})\phi_2\}\xi_{01}. \end{aligned} \tag{17}$$

This is readily recast into

$$\begin{aligned} x_2&=x_1[\phi_1, -d_{12}/n_{12}]+\xi_{01}[-d_{12}/n_{12}],\\ \xi_2'&=x_1[\phi_1, -d_{12}/n_{12}, \phi_2]+\xi_{01}[-d_{12}/n_{12}, \phi_2]. \end{aligned} \tag{18}$$

The coefficients a_2 and c_2 of x_2, and b_2 and d_2 of ξ_2' are seen to be evaluations of the Gaussian brackets:

$$\begin{aligned} a_2&=[\phi_1, -d_{12}/n_{12}], & b_2&=[-d_{12}/n_{12}],\\ c_2&=[\phi_1, -d_{12}/n_{12}, \phi_2], & d_2&=[-d_{12}/n_{12}, \phi_2]. \end{aligned} \tag{19}$$

It can be easily proved that if

$$\begin{aligned} x' &= x_1[\phi_1, -d_{12}/n_{12}, \cdots -d_{k-1,k}/n_{k-1,k}] \\ &\quad + \xi_{01}[-d_{12}/n_{12}, \phi_2, \cdots -d_{k-1,k}/n_{k-1,k}], \\ \xi' &= x_1[\phi_1, \cdots -d_{k-1,k}/n_{k-1,k}, \phi_k] \\ &\quad + \xi_{01}[-d_{12}/n_{12}, \cdots \phi_k] \end{aligned} \tag{20}$$

is true for $(k-1)$ surfaces, then it must be true for k surfaces.

The coefficients satisfy Eq. (2), since the Gaussian brackets satisfy Eq. (15).

To use Eqs. (1) in practical problems, the surface powers and negative optical separations are arranged in a series in the order in which they occur in the system. Then, in evaluating these as Gaussian brackets, the first surface power is omitted for d, the last power for a, the first and the last powers for b, and nothing is omitted for c. As just shown, the power of the entire system is then c, its back focus is a/c, and its front focus is $-d/c$. An object at distance s from the first vertex is imaged at distance s' from the last surface, where s' is given by (6) as a function of s, or by (7) as a function of the magnification m. Focal length and back focus are given by (3) and (4).

In computations where lens thicknesses can be neglected, the terms representing them reduce to zero and, in accordance with I, the three members representing each of the two surface powers and the intervening optical distance can be combined into a single member, the sum of the two surface powers. The series from which the brackets are constructed is then composed of alternate lens powers and separations. If we call e the separation and

$$\psi = (n-1)(1/r_A - 1/r_B) \tag{21}$$

the power of a thin lens, we obtain

$$\begin{aligned} a &= [\psi_1, -e_{12}, \cdots -e_{k-1,k}], \\ b &= [-e_{12}, \psi_2, \cdots -e_{k-1,k}], \\ c &= [\psi_1, -e_{12}, \cdots \psi_k], \\ d &= [-e_{12}, \cdots \psi_k]. \end{aligned} \tag{22}$$

A plane-refracting surface in a system represents zero surface power. Here again, in accordance with I, three members of the series can be combined into a single member, the sum of the optical distances on either side of the plane to the adjacent surfaces.

EXAMPLES

As an example, let us assume that we want to change one of the construction elements of a given system of thin lenses with finite distances, but want to keep the focal length constant by compensating with (a) the last distance or (b) the power of the last lens. We want to know, first, how to change this correction element if the change of the first element is given. We want to know, second, how the back focus changes under these circumstances.

Equations (3), (4), and (22) give

$$\begin{aligned} \Phi &= [\psi_1, -e_{12}, \psi_2, \cdots \psi_k], \\ s_F'\Phi &= a = [\psi_1, -e_{12}, \cdots -e_{k-1,k}], \end{aligned} \tag{23}$$

where Φ is the power of the full system.

From (23) we find

$$\Phi = [\psi_1, -e_{12}, \cdots -e_{k-1,k}]\psi_k + [\psi_1, -e_{12}, \cdots \psi_{k-1}],$$

or

$$\begin{aligned} \psi_k &= \frac{\Phi - [\psi_1, \cdots \psi_{k-1}]}{[\psi_1, \cdots -e_{k-1,k}]}, \\ s'_F\Phi &= [\psi_1, \cdots -e_{k-1,k}]. \end{aligned} \tag{24}$$

Equation (24) solves the problem if we compensate with the last power. For instance, let us assume that the power of the second lens is changed and compensate with the last. To investigate this, we simply separate (24) as linear functions of ψ_2 according to (13), and find

$$\begin{aligned} \psi_k &= \frac{\Phi - \psi_2[\psi_1, -e_{12}][-e_{23}, \cdots \psi_{k-1}] - [\psi_1, -(e_{12}+e_{23}), \psi_3, \cdots \psi_k]}{\psi_2[\psi_1, -e_{12}][-e_{23}, \cdots -e_{k-1,k}] + [\psi_1, -(e_{12}+e_{23}), \cdots -e_{k-1,k}]}, \\ s'_F\Phi &= \psi_2[\psi_1, -e_{12}][-e_{23}, \cdots -e_{k-1,k}] + [\psi_1, -(e_{12}+e_{23}), \cdots -e_{k-1,k}]. \end{aligned} \tag{25}$$

As a second example, we shall change the first distance and compensate with the last

distance. Equation (23) gives

$$\Phi = -e_{k-1,k}\psi_k[\psi_1, \cdots \psi_{k-1}]+[\psi_1, \cdots (\psi_{k-1}+\psi_k)],$$

$$e_{k-1,k}=\frac{[\psi_1, \cdots (\psi_{k-1}+\psi_k)]-\Phi}{\psi_k[\psi_1, \cdots \psi_{k-1}]}, \quad (26a)$$

$$s'_F\Phi\psi_k=\Phi-[\psi_1, \cdots \psi_{k-1}]. \quad (26b)$$

Formulae (26) show how any change in the system may be compensated by a change of $e_{k-1,k}$.

For instance, we find for a change of e_{12}:

$$e_{k-1,k}=\frac{\begin{array}{r}\psi_1 e_{12}[\psi_2, \cdots (\psi_{k-1}+\psi_k)] \\ +\Phi-[(\psi_1+\psi_2), \cdots \psi_{k-1}]\end{array}}{\begin{array}{r}\psi_k\psi_1 e_{12}[\psi_2, \cdots \psi_{k-1}] \\ -\psi_k[(\psi_1+\psi_2), \cdots \psi_{k-1}]\end{array}},$$

$$s'_F=\frac{\begin{array}{r}\psi_1 e_{12}[\psi_2, \cdots \psi_{k-1}]+\Phi \\ -[(\psi_1+\psi_2), \cdots \psi_{k-1}]\end{array}}{\Phi\psi_k}. \quad (27)$$

Let us specialize for the triplet ($k=3$). We then find explicitly instead of (25)

$$\psi_3=\frac{\psi_2(1-e_{12}\psi_1)-(\Phi-\psi_1-\psi_3+(e_{12}+e_{23})\psi_1\psi_3)}{\psi_2(1-e_{12}\psi_1)e_{23}+(1-(e_{12}+e_{23})\psi_1)}, \quad (25c)$$

$$s'_F=-(1/\Phi)\{e_{23}\psi_2(1-e_{12}\psi_1)+(1-(e_{12}+e_{23})\psi_1)\};$$

and instead of (27)

$$e_{23}=\frac{e_{12}\psi_1(\psi_2+\psi_3)-\psi_1-\psi_2+\Phi}{\psi_3(e_{12}\psi_1\psi_2-\psi_1-\psi_2)},$$

$$s'_F=\frac{e_{12}\psi_1\psi_2-\psi_1-\psi_2+\Phi}{\Phi\psi_3}. \quad (27c)$$

GAUSSIAN BRACKETS IN FINITE OPTICS

In tracing finite rays (not necessarily in the meridian plane), we obtain expressions which are very closely related to those developed in the first part of the paper. Let x_λ, y_λ be the coordinates of the intersection point of a ray with the λth surface. Let $\xi_{\lambda,\lambda+1}$, $\eta_{\lambda,\lambda+1}$ be the direction cosines of the ray between the λth surface and the $(\lambda+1)$st surface, multiplied by the refractive index.

In analogy with (10), we then find the equations:

$$\begin{array}{l}\xi_{12}=\xi_{01}+\phi_1 x_1, \\ \eta_{12}=\eta_{01}+\phi_1 y_1, \\ \qquad\text{(first refraction)} \\ x_2=x_1-(\delta_{12}/n_{12})\xi_{12}, \\ y_2=y_1-(\delta_{12}/n_{12})\eta_{12}, \\ \qquad\text{(first transposition)} \\ \xi_{23}=\xi_{12}+\phi_2 x_2, \\ \eta_{23}=\eta_{12}+\phi_2 y_2. \\ \qquad\text{(second refraction)}\end{array} \quad (28)$$

The power ϕ is here given by

$$\phi_\lambda=(n_{\lambda,\lambda+1}\cos\epsilon_\lambda'-n_{\lambda-1,\lambda}\cos\epsilon_\lambda)/r_\lambda, \quad (29)$$

where ϵ and ϵ' are incident and refracted angles, respectively, and the δ's are the distances between consecutive surfaces measured along the ray.

If we eliminate the intermediate steps, we arrive again at formulae (1) and (20), that is:

$$\begin{array}{l}x'=ax+b\xi, \quad y'=ay+b\eta, \\ \xi'=cx+d\xi, \quad \eta'=cy+d\eta; \\ a=[\phi_1, \cdots -\delta_{k-1,k}/n_{k-1,k}], \\ b=[-\delta_{12}/n_{12}, \cdots -\delta_{k-1,k}/n_{k-1,k}], \\ c=[\phi_1, \cdots \phi_k], \quad d=[-\delta_{12}/n_{12}, \cdots \phi_k];\end{array} \quad (30)$$

where, of course, the ϕ's and δ's change from ray to ray.

Previous studies showed the relations between a, b, c, d, and the image errors.[3, 4]

Since the δ's give the distances between consecutive surfaces on the ray and the ϕ's are measures of the angular deviations at the surfaces, formulae (21) show how the geometrical configuration of the system influences the image. This will be discussed in a forthcoming paper.

A Compendium of Line Pairs and Operating Conditions Used in Spectrographic Analysis*

EDWIN S. HODGE
Harry W. Dietert Company, Detroit, Michigan

INTEREST in and application of spectrochemical methods have greatly expanded in the last few years. With this growth there has been a large influx of new workers into the field, some of whom have had little or no previous training or experience in spectrographic analysis.

In working out methods for quantitative analysis, this group, as well as those who are more experienced, frequently ask what line pairs and operating conditions should be used. The answer usually is to experiment and determine by trial and error what these optimum conditions are. Some few have access to the experience of others either through training or description of accepted methods when they are available.

The present paper is a preliminary report of a compilation of information from most of the available literature. Its purpose is to put in simple concise form most of the operating conditions which have been reported for modern quantitative spectrographic analysis. It is felt that such a compilation might help in the current trend to standardize present spectrographic procedures.

In setting out to abstract systematically the literature in this field, a definite plan is required. This is especially true in the case where many various methods have been used. In recent years, however, the trend has been towards the almost universal adoption of the internal standard method first proposed by Gerlach.

The intensities of the unknown element line and base element line are measured by means of a log sector or microphotometer and are plotted as a ratio against the concentration of the unknown element present in the standard. This scheme has the advantage that operating conditions are more accurate and reproducible than any other widely used method of spectrographic analysis. In view of this trend the application of the internal standard technique and the listing of the line pairs used have been set up as the first criterion for abstracting a reported procedure.

It is also required that the concentration range covered be given, and this is considered as a second criterion. This is given as percent, atomic percent, milligrams per 100 milliliters, milligrams per milliliter, or some other concentration parameter depending upon the purpose of the investigation.

The type of source used for excitation will also be a deciding factor in determining the sensitivity and nature of the spectra that will be emitted. Therefore, it is considered important that the excitation source used by the writers should be stated.

These rules have been the factors used to determine whether the information given in the papers examined was to be included in this work. They have been rather liberally interpreted. For example, Lundegardh's technique has been included, since this involves a modification of the internal standard method by using the unknown line *vs.* the background nearby. The procedure using homologous pairs has also been included.

The preparation of this collection has been based on the use of the *Index to the Literature on Spectrochemical Analysis* by Meggers and Scribner.[1] This bibliography, published by the A.S.T.M., lists all the articles on spectrochemical analysis published between 1920 and 1939 which were known to the authors. These articles are listed by years, authors, title of the paper, the original journal, and the *Chemical Abstract* or similar reference, when it is available. It is expected that the present compilation and the Index are to be used in conjunction with each other. Permission from the authors for its use in this connection is gratefully acknowledged.

All 1500 of these references have been examined by title, abstract reference, or the original article, to determine whether they contain infor-

* Paper presented at the Twenty-Eighth Annual Meeting of the Optical Society of America, Pittsburgh, Pennsylvania, October 7–9, 1943.

[1] W. F. Meggers and B. F. Scribner, *Index to the Literature on Spectrochemical Analysis, 1920–1939* (American Society for Testing Materials), second edition.

TABLE I. Table of operating conditions for magnesium analysis.

Meggers No. or ref.	1085	1376	A-1	A-2	A-3	A-4	A-5
Instrument	S-2	S-2	S-3	S-1		S-1	S-1
Source	Spark	Spark	Feussner spark	Spark (controlled)	Spark*	Spark (controlled)	a.c. arc
Voltage	.../20,000 v	.../20,000 v	.../12,900 v	230/35,000 v	.../15,000 v	230/35,000 v	2.5 kv
Current or power			75 ohms in primary	4/3 kw	½ kw	½ kw	3.8 amp.
Inductance	0.25 mh	0.25 mh	0.08 mh	1.4 mh	0.05 mh	1.4 mh	
Capacitance	0.005 μf	0.005 μf	0.0007 μf	0.014 μf	0.005 μf	0.007 μf	
Source gap	5 mm		2 mm	3 mm	3 mm	3.0 mm	3 mm
Electrode (upper)	Electrodes 1 cm× 1 mm	Sample area rectangular	Cast Mg rods cut down to	Mg disk cast from super heated metal	Cast Mg and pin electrodes cut	Cast Mg disk	Cast Mg disk
Electrode (lower)		Mg. electrode Area 1×10 mm	4 mm diam.	Pointed carbon electrode	on special cutter	Tapered carbon electrode	Tapered carbon electrode
Slit width	0.01 mm		0.03 mm	0.080 mm	(1) 0.05 mm	0.080 mm	0.05 mm
Pre-burn/exp/sector (Time in seconds)	30/varied exp/	"Variant exposure" method	30/30 to 90/	10/50/325 mesh screen	30/30/	10/10/	15/15/
Emulsion	E-2	E-2 and 3	E-4	E-1	E-2	E-1	E-1
Developer	M.Q.		D-1	D-1 diluted 1-2 pt.	D-2	D-1	D-1
Time...Temp.	3'—65°F		3'—65°F	1½'—70°F H_2O	4½'—67°±1°F	3'—65°±1°F	2 min. 70°F
Calibration used				Yes		Yes	Yes
Lens used	Sphereo—cyl. lens used	No lens	2 lens system to focus on the plate	Yes (2)		Yes	Yes Grating aperture set to 7

Remarks: (See remarks under Table II.)
* Auxiliary gap in series used in circuit.

References:
A-1 Metal Industry (London) **60**, 140-3, 159-63 (1941).
A-2 Bohn Alum. and Brass Co., J. LaMotte Contributed Data.
A-3 J. Soc. Chem. Ind. **60**, 278-86 (1941).
A-4 Alum. Co. of Am. Methods 143 and 743.
A-5 Wright Aero. Corp. D. L. Garland, Con'd. Data.

Spectrographs:
S-1 ARL-Dietert Grating.
S-2 Hilger Med. Quartz.
S-3 Zeiss Med. Quartz.

Emulsions:
E-1 Eastman S. A. No. 1 Film.
E-2 Ilford Ordinary.
E-3 Ilford Special Rapid Pan.
E-4 Ilford Half Tone.

Developers:
D-1 Eastman D-19.
D-2 Ilford D-2.

TABLE II. Table of line pairs for magnesium analysis.

Unk. line	Ref. line	Range	Source (6)	Ref.
Al 3944.0	Mg 3074.0 (3)	0.5%* (4)	S	A-4
2567.9	2915.5	1.0 –10.0%	S	1085
3082.1	2915.5	1.0 – 9.0%	S	A-1
2567.9	2915.5	4.0 –10.5%	S	1376
2567.9	2779.8	5.3 – 6.7%	S	A-2
2660.4	3074.0	6.5%*	S	A-4
2567.9	3074.0	7.0%*	S	A-4
2652.5	3074.0	11.5%*	S	A-4
Ca 3179.3	Mg 3074.0	0.025– 0.25%	S	A-3
Cu 3274.0	Mg 3074.0	0.008– 0.3%	S	A-3
3247.6	2779.8	0.01 – 0.15%	S	A-2
3274.0	2915.5	0.01 – 0.1%	S	1085
3274.0	2915.5	(5) **0.1%	S	1376
3247.6	3074.0	0.02%*	S	A-4
3274.0	3074.0	0.04%*	S	A-4
Fe 2483.3	2695.2	0.001– 0.05%	ACA	A-5
Mn 2949.2	Mg 2915.5	0.001– 0.5%	S	A-1
2605.6	3074.0	0.04 – 0.35%	S	A-3
2605.6	2915.5	**0.5%	S	1376
2610.2	2915.5	0.05 – 0.5%	S	1376
2949.2	2779.8	0.10 – 0.3%	S	A-2
2939.3	2915.5	0.25 – 2.5%	S	1376
2949.2	3074.0	0.25%*	S	A-4
3441.9	3074.0	0.36 – 1.7%	S	A-3
3228.1	3074.0	1.6%*	S	A-4
Ni 3414.8	2698.1	0.001– 0.05%	ACA	A-5
3002.5	2698.1	**0.05%	ACA	A-5
Pb 2614.2	2698.1	0.01 – 0.15%	ACA	A-5
Si 2516.1	2698.1	0.002– 0.10%	ACA	A-5
2516.1	3074.0	0.01 – 0.35%	S	A-3
2516.1	2779.8	**0.2%	S	A-2
2881.6	3074.0	0.17%*	S	A-4
	Narrower slit may be required			
Sn 3175.0	2698.1	0.001– 0.05%	ACA	A-5
2706.5	2698.1	**0.05%	ACA	A-5
Zn 3345.0	2915.5	0.1 – 1.0%	S	1376
3345.0	3074.0	0.1 – 2.9%	S	A-3
3302.9	2915.5	0.2 – 1.0%	S	1085
3302.9	3074.0	1.0%*	S	A-4
3282.3	3074.0	2.0%*	S	A-4
3282.3	2779.8	2.5 – 3.5%	S	A-2

Remarks:
(1) A radial sector or a screen is used for reducing the intensity.
(2) A No. 2 microscope cover glass used for a filter for Mg quintuplet. When Mg 2779.8/Mg 3074.0 intensity ratio is outside 0.94–0.98 the Zn and Al results were in wide error.
(3) Other lines could be used for internal standard line, e.g., 2778.3, 2781.4, 2783.0, and 2915.5. For the latter a microscope cover glass is necessary. Of course the E.P. value will vary then in either case.
(4) Term E.P. or equivalence point is applied to the conc. of the unknown element which will produce a ratio Ix/I Mg = 1.0 under conditions given. To conserve space in these tables this is marked by an * following the percent sign.
(5) Some papers indicate the upper limits of their working ranges. The values marked ** are to be considered up to the percent given. In general such ranges will generally be useful over a tenfold concentration range.
(6) S = Spark; ACA = Alternating current, high voltage arc.

mation falling within the conditions outlined above. Articles dealing with such subjects as instrument design, qualitative identification, spectral terms, wave-length tables, etc., were easily eliminated by title. Others dealing with spectral analysis and similar topics were examined in *Chemical Abstracts* to determine if the original article should be consulted. Frequently, these references could be eliminated when the abstract mentioned that the quantitative analysis was made by visually comparing the sample to a series of independent standards photographed at the same time. All other articles, approximately 400, were noted and examined at the original source if possible. Frequently these used comparison methods and hence their data could not be recorded. In order to speed up the completion of the work, articles written in Dutch, Hungarian, Scandanavian, Japanese, and Russian have been omitted from the current examination.

During the review of the original papers, the line pairs and the concentration ranges covered were recorded with references on small filing cards of various colors for the different types of alloys or analyses being studied. All these recorded lines have been divided into tables for

TABLE III. Table for data contribution to spectrographic analysis compendium.

I.	II.	III.	Unk. line	Ref. line	Range	Method I, II, or III
Instrument						
Source						
Voltages						
Current or kw						
Inductance						
Capacitance						
Gap (mm)						
Upper electrode						
Lower electrode						
Slit width (mm)						
Pre-burn/exp/sector						
Emulsion						
Developer						
Dev. time—Temp.						
Cal. used						
Lens used and how						
Remarks:						

I, II, and III are given for varied conditions that may be used. Return to: Edwin S. Hodge
Harry W. Dietert Company
9330 Roselawn Avenue
Detroit 4, Michigan

Signed:_____

Company:_____

Position:_____

steels, aluminum, magnesium, brass, tin, lead, other non-ferrous metals, minerals and oxides, biological materials, and other miscellaneous types of samples.

At the same time the information on line pairs was collected, data concerning the operating conditions were sought. This information was recorded under the following items:

1. Meggers and Scribner's reference number or other reference.
2. Instrument used.
3. Type of source used.
4. Voltage (ratings on transformer if given).
5. Current or power.
6. Inductance (mh).
7. Capacitance (μf).
8. Source gap (mm).
9. Upper electrode material
10. Lower electrode material
11. Slit width (mm).
12. Exposure expressed as: preburning of source (seconds), exposure (seconds), radial sector opening or screen if used.
13. Photographic emulsion.
14. Developer.
15. Developing Time (minutes)—Temp. (°F or °C).
16. Calibration method used.
17. Buffer used.
18. Lens used, and how.
19. Other remarks—buffers—sample preparation, etc.

In order to illustrate the information collected in this survey, some of the more important and recent data on the analysis of magnesium have been put in a form similar to that to be used in the final report. This is shown in Table I for the operating conditions and Table II for the line pairs. It should be noted that this table is merely illustrative and not complete.

It must be emphasized that these data are not intended to be used for setting up a working procedure without testing the information with the particular operating circumstances at hand. Rather, the data are meant to point out line pairs and conditions that other workers have found to be satisfactory for *their* purposes, and consequently are given *only as possible suggestions* for making satisfactory working curves and analyses. It should be pointed out that frequently the presence of some other factor in the sample such as heat treatment, extraneous elements, and difference in sample preparation, etc., may present difficulties making a previously successful technique of very little value. These factors should be considered in developing a given procedure.

So far no serious effort has been made to

collect unpublished information from laboratories. Nevertheless, to make the data of the most use to spectrographers as a whole, it would be very desirable to include as much of this unpublished information as possible. To facilitate the collection of this material Table III shows blanks for the type of information on any type of emission analysis that is desired. Any original contributions which workers wish to contribute will be gratefully received and duly acknowledged in the final publication. January 31, 1944 has been set as a time limit for receipt of such data.

A survey such as this has given the writer ample opportunity to observe that many good papers, as judged by the quantitative results, are of little value because the important operating conditions have been omitted. Several well-written papers showing excellent agreement of results left out all mention of the lines used or the concentrations that could be covered. Others stated that inductance and capacitance were used but the values were omitted. It is suggested that those submitting papers for publication use the above list to determine if the data are complete. While many items are not highly important, for example, developing time, temperature, etc., everything has been included for possible comparison purposes.

Grateful acknowledgment is made for contributions of information from Messrs. J. R. Churchill, Aluminum Company of America, John La Motte, Bohn Aluminum and Brass Corporation, and D. L. Garland, Wright Aeronautical Corporation.

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 33, NUMBER 12 DECEMBER, 1943

Spectographic Determination of Thorium in Tungsten Filament Wire*

S. L. PARSONS
Sylvania Electric Products Inc., Emporium, Pennsylvania

IT is well known that a monatomic layer of thorium over the surface of a tungsten filament lowers the work function of tungsten and greatly increases the thermionic emission. In the manufacture of transmitting tubes, which takes advantage of this enhancement, it is necessary to set up rather lengthy and critical processing schedules in order to secure the proper layer of thorium essential for good performance. These schedules are determined by making a number of experimental tubes and are dependent upon the amount of thorium present in the filament wire. The filaments are processed before assembly in the tube and since it is rather difficult to reprocess a filament which does not have adequate emission, a variation of thorium content can cause the loss of a valuable product. Chemical analysis of the wire is possible; however, the difficulty of putting tungsten rapidly into solution rules it out as a means of control. The spectrograph now has a chance to be of considerable service in the routine control of the wire if a procedure having sufficient sensitivity and accuracy can be worked out.

Our spectrographic section was asked to develop a method for determining thorium in tungsten over the range from 1.0 percent to 2.0 percent. A preliminary survey was made using the d.c. arc to see if the program looked feasible. The tungsten spectrum consists of 4327 lines between 10,000 and 2000 angstroms[1] so some difficulty, due to interference between tungsten and thorium lines, was expected. We found that our Bausch & Lomb large Littrow spectrograph had enough dispersion in the ultraviolet, where a number of thorium lines are available, to give us several analysis pairs. After trying a number of them, we settled on thorium 2899.3A and tungsten 2904.0 as being most suitable under our exposure conditions.

A series of sample wires 0.013″ diameter drawn

* Paper presented at the Twenty-Eighth Annual Meeting of the Optical Society of America, Pittsburgh, Pennsylvania, October 7–9, 1943.

[1] *Massachusetts Institute of Technology Wavelength Tables* (John Wiley & Sons, Inc., New York, 1939).

from seven melts of tungsten containing varying amounts of thorium, was obtained. The samples from these melts were chemically analyzed and were then used for standards in the preparation of an analytical curve. The curve shown in Fig. 1 is of the usual form relating the logarithm of the ratio of the intensity of the thorium line 2899.3 to the intensity of the tungsten line 2904.0, and the concentration. The same procedure used for determining the log I ratio in

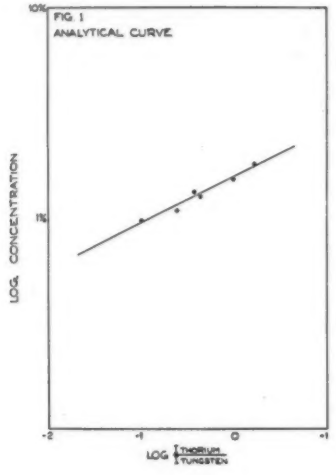


FIG. 1.

setting up the analytical curve is carried out for every set of samples. These log I values are obtained from a plate calibration curve, relating blackening and log intensity. Eastman type 33 plates are used and a calibration curve is drawn for each plate. To obtain this curve, a standard iron exposure is run with every set of samples, ten selected iron lines are then read and their blackenings are plotted against the log I value assigned each line from measurements made with a rotating step sector. This is a well-known method of plate calibration as described by Sawyer and Vincent[2] and has worked very well. The analytical curve has been used over the range from 0.75 percent to 2.25 percent, which more than fulfills the original requirements.

[2] "Developments in the Rapid Spectrographic Analysis of Cast Iron," *Proceedings of the Fifth Summer Conference*, p. 1.

As soon as we started investigating the repeatability of the analysis procedure, we found our main problem would be to get the required accuracy. The thorium content of good filament wire runs about 1.5 percent and the Power Tube Group wished to set up specifications which would hold the wire to limits of 1.45 to 1.55 percent, or a variation of ± 3.5 percent. This accuracy demand immediately rules out the use of the d.c. arc and indicated the use of the a.c. arc or high voltage spark with most of the emphasis on the latter. As our laboratory is not equipped with a high voltage interrupted spark source, we were forced to confine our attentions to the a.c. arc. Our arc supply consists of a power transformer having a 220-volt primary and four 1100-volt secondaries, each capable of supplying 1.5 amperes. A switching system allows the secondaries to be hooked in series, parallel, or series-parallel, so that three voltage ranges are possible: 1100, 2200, and 4400. Suitable ballast resistors in the secondaries permit one to control the current from 0.75 ampere to 6 amperes.

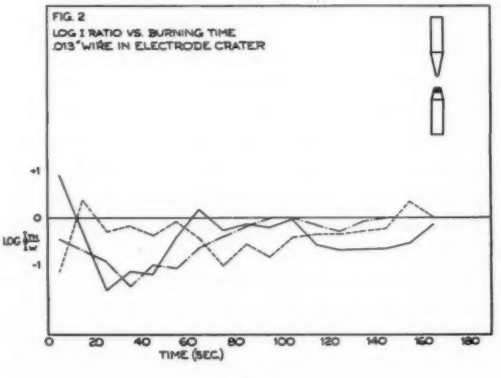


FIG. 2.

After investigating a large number of current and voltage combinations, we settled on 2200 volts, 2.4 amperes as being most suitable.

A few measurements on the four main sources of error in spectrographic analysis showed us that most of our trouble was due to variations in the light source and to sample inhomogeneity. A study was made to see if we could either improve the method of getting the material into the arc or if we could find arcing conditions which

would be more reproducible. Although it appeared preferable to use some form of electrode which would not require too much time in preparing the wire sample for arcing, the possibility of chemically dissolving the wire, evaporating the solution onto carbon rods, and arcing the prepared rods was investigated. Samples of wire were electrolytically dissolved in sodium nitrite solution using about 35 volts a.c. The solution was evaporated to dryness on the flat ends of two prepared graphite rods which were then arced. It was hoped that the method would prove feasible as it provided a means of preparing standards to check our chemically analyzed wire. Unfortunately, however, we found it necessary to use the d.c. arc in order to have adequate sensitivity and the repeatability of this arc was not good enough for our problem. Our next attempt was then to use samples of the wire coiled up and inserted in the crater in the lower graphite electrode while a tapered graphite electrode was used on top. With this arrangement repeat plates gave us standard deviations of from 13 to 15 percent. In order to find out more about this arc, we prepared single samples of 1.5 percent thoriated wire and burned them continuously for three minutes taking five-second exposures every five seconds. Plotting the log I ratio of the thorium line to the tungsten line against time gives the curve shown in Fig. 2. This curve is a plot of a set of three independent runs and shows that at the beginning of the arc the log I ratio varies considerably while after a period of time the ratio approaches a more constant level. It is evident that a preburn time of about eighty seconds should get rid of some of the experienced variation. Repeat tests on a single sample showed standard deviations of from 5.5 percent to 8.6 percent after the preburn was used.

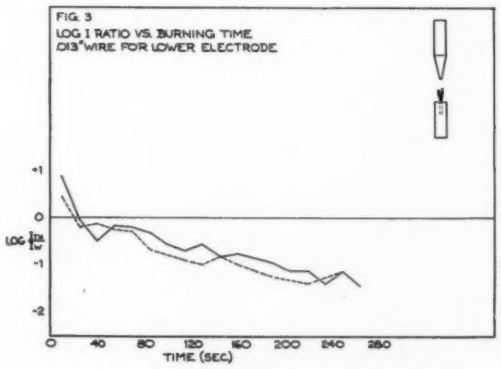


FIG. 3.

About this time a slightly different form of electrode was tried out. The end of a $\frac{1}{4}''$ graphite electrode was cut off square and a small hole (0.042" diameter) was drilled in an end for a depth of about $\frac{1}{4}''$. Several pieces of wire were inserted in this hole and cut off so that they extended about $\frac{3}{32}''$ above the carbon. This bundle of wires was then used as the lower electrode, while a tapered graphite rod was used for the upper electrode. The arc struck directly to the tungsten wire from the upper graphite. A sample prepared in this manner and burned continuously for four minutes while a series of exposures were made, gave the curve shown in Fig. 3. Here we have considerable change; the log I ratio does not fluctuate as badly with burning time, and a new effect enters in. As the sample is burned, the log I ratio goes more and more negative, indicating that the thorium in the sample burns out at a fairly uniform rate after about the first minute. Attempts to use this arc gave more repeatable results than our previous electrode form; however, there was still too much source variation to allow accuracies comparable with those required.

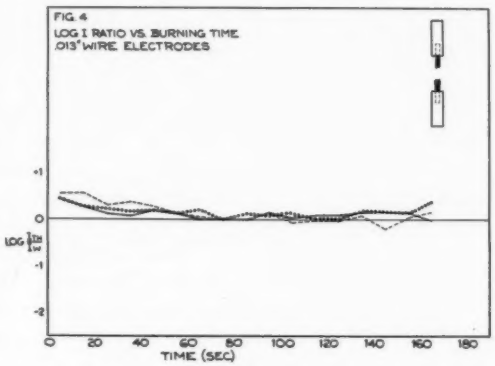


FIG. 4.

The next step was to make both electrodes by drilling into carbon rods and packing short lengths of wire into each. The burning characteristic of this arc is shown in Fig. 4. Here we have

a still further improved source. There is a little variation at the beginning of the arc and a tendency for the thorium to burn out, but after about a minute's time the log I ratio becomes fairly constant and continues so for another minute and a quarter, when it again becomes somewhat erratic. The variation at the end of the burn occurs when the wires have burned down to a point where the arc plays to the carbon rods. With this light source, our first set of twelve determinations on a single sample, gave a standard deviation from the mean of 2.78 percent using a preburn of fifty seconds and a sixty-second exposure chosen according to Fig. 4.

In order to satisfy ourselves completely as to the reliability of the procedure, a set of 84 samples was taken from a coil of wire whose chemical analysis showed 1.5 percent thorium. The exposures were made on seven plates and the standard deviation from the mean was found to be 3.54 percent. The maximum variation was ± 5.3 percent, which means that possibly 30 percent of our determinations would vary more than the specifications set up by the Power Tube group would allow. Upon running analyses on various melts of tungsten wire, we found that the thorium content in successive spools of wire drawn from the same melt varied more than the maximum deviation of the analysis procedure. For this reason, the Power Tube group relaxed its specifications and the procedure was put into routine use.

After we had been running routine control analyses for a while, we were asked to extend the method to a number of different size filament wires varying from 0.013″ to 0.004″. All our original work had been done on the larger diameter wire. In order to avoid having a series of wires chemically analyzed so that analytical curves could be built up for every wire diameter used, we looked into the possibility of using our 0.013″ analytical curve for all sizes. Electrodes, prepared as previously described, using the various wire sizes still gave standard deviations of about 3.5 percent. A sample of each of the various diameters was chemically analyzed and then checked spectrographically using the analytical curve shown in Fig. 1. The agreement was very good, in every case the discrepancy was less than 5 percent, so the procedure was extended to all sizes between 0.004″ and 0.013″. The most difficulty experienced has been in loading the finer wires. We now analyze every spool of wire before it is put into manufacture. The result of each such analysis is combined with that of a metallurgical examination so that the filament wire is well inspected before it is used. In a period of over a year during which this procedure has been employed, over 1000 samples have been analyzed with very satisfactory results.

I am greatly indebted to Mr. L. D. Caskey (now in the Army Air Corps), who made many of the measurements necessary for this development, and to Mr. T. J. Organ who has been connected with it during its latter stages, for their enthusiasm and assistance. Without their contributions the problem would not be the smooth running routine control it is now.

Hydrogen Discharge Tube with Metal Capillary

EGON LORENZ AND PAUL D. KUECK
National Cancer Institute, Bethesda, Maryland

(Received October 21, 1943)

THE most convenient source for absorption studies in the ultraviolet is the continuous spectrum of molecular hydrogen. During the last 15 years various types of hydrogen discharge tubes have been described in the literature; a list of references to 1938 is given by Noyes L. Smith.[1] A hydrogen arc of high intensity was described by Allen[2] in 1941. The type of tube used in this investigation follows the usual construction. It consists of 2 cylindrical aluminum electrodes enclosed in glass cylinders. These cylinders are connected by a capillary of several cm length and a cross section of several mm². The electrodes carry holes for the passage of the discharge and the radiation emitted by the capillary. The ultraviolet radiation emerges through a quartz window cemented to a side tube of one of the glass cylinders. The lamp is operated with a.c. For cooling purposes it is submerged in water. If a glass capillary is used, it is usually silvered or platinized to favor recombination.

The glass capillary has several disadvantages. It is fragile, since its walls must be thin for adequate cooling. The current which can be passed through the capillary in continuous operation is limited, as destruction of the glass will result if the temperature gradient in the wall of the capillary becomes too high, and finally with too high a temperature at the inner wall of the capillary the ultraviolet output will decrease since recombination is slower with increasing temperature. These disadvantages can be avoided by replacing the glass capillary with a metal capillary (e.g., copper) and joining it to the glass tubes, carrying the aluminum electrodes, by metal-glass seals as shown in Figs. 1a and 1b. Metal capillaries have been used previously in discharge tubes for the production of proton beams.[3] If a hydrogen discharge tube constructed with such a metal capillary is connected to approximately 1000 volts a.c. and the hydrogen pressure slowly raised beginning with pressures below 1 mm of Hg, the following observations are made: For pressures below 1 mm the capillary will be visible as a dark opening through the luminous discharge covering the front of the capillary tube and filling the glass tubes. At a pressure of approximately 1 mm occasional bright flashes will go through the capillary increasing in frequency with increasing pressure, and at about 1.5 mm the discharge through the capillary has become continuous and increases in intensity with increasing pressure. At still higher pressures (10 mm) the discharge becomes unsteady, the intensity of the capillary discharge has decreased considerably. During degassing

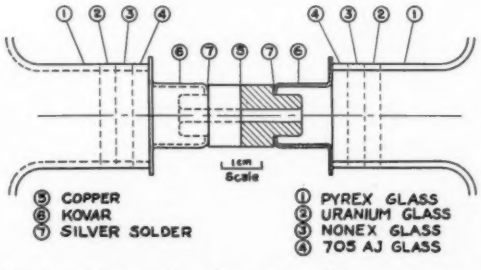


FIG. 1a. Construction of metal capillary and glass to metal seal.

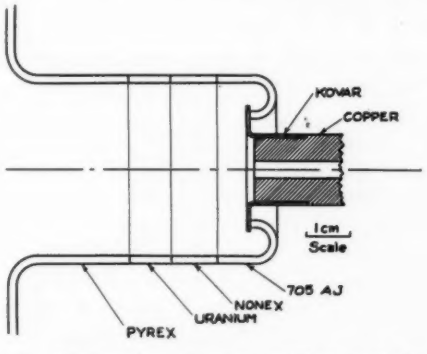


FIG. 1b. Construction of metal capillary and glass to metal seal.

[1] Noyes L. Smith, J. Opt. Soc. Am. **28**, 40 (1938).
[2] Alexander J. Allen, J. Opt. Soc. Am. **31**, 268 (1941).
[3] E. S. Lamar, W. W. Buechner, and R. J. Van de Graaf, J. App. Phys. **12**, 132 (1941).

strong copper lines appear, disappearing after the degassing is finished with the exception of $\lambda = 3248$A and $\lambda = 3274$A which remain as very sharp lines just observable above the continuum. The ultraviolet continuum emitted from the copper capillary is identical with that emitted from a glass capillary with regard to spectral distribution and intensity. The phenomenon of the discharge going through the capillary rather than through the copper can perhaps be explained

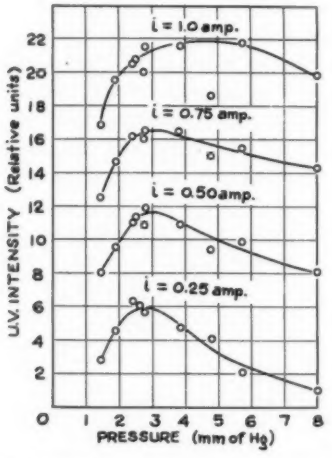


FIG. 2. Ultraviolet intensity dependent upon pressure for various arc currents.

by the fact that the cathode drop on that face of the capillary nearest to the electrode which is anode at the moment is so high because of its small surface that the path through the capillary is the path of the least resistance.

Figures 1a and 1b show constructions of the capillary with the glass to metal seal. The simpler construction is that of Fig. 1a, as far as the glass to metal seal is concerned, and will be sufficient for most purposes. For lamps to be run continuously at high amperage, the seal of Fig. 1b is preferable as it gives a better protection of the seal from the discharge. The capillary of Fig. 1a has a diameter of 3 mm and is 3 cm long. Longer capillaries can be used and will give a gain in intensity.[4] It has to be considered, however, that with increasing length the useful field of even illumination will become more and more restricted. Several such lamps were built and run at various currents up to 2 amp. One lamp had been in service for several hundred hours before refilling became necessary. It was run at 800 milliamp. One lamp constructed according to Fig. 1b[5] with a diameter of the capillary of 4 mm and a length of 4 cm was operated at currents of 5–6 amp. The electrodes consisted of corrugated aluminum sheet to prevent an anomalous cathode drop at high current densities.

Figure 2 gives the ultraviolet intensity in relative units depending upon hydrogen pressure for various tube currents. The ultraviolet intensity was measured with a tantalum high vacuum photo-cell in ultraviolet transmitting glass. The long wave limit of this cell is at approximately 3000A. The short wave cut off of the glass lies at approximately 2100A. The photocurrent was amplified by a type 954 acorn tube and measured with a microammeter in the plate circuit. The figure shows a maximum value for the ultraviolet output for pressures between 2 and 5 mm Hg, the maximum shifting to higher pressures for larger currents. The ultraviolet

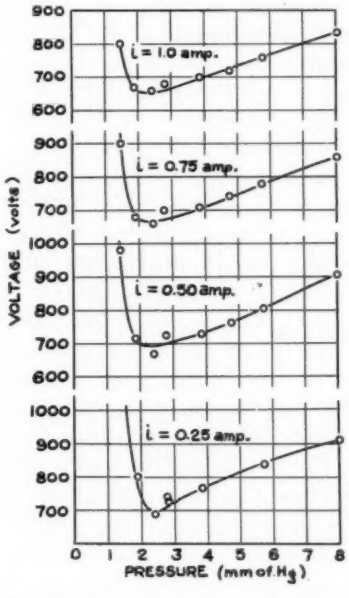


FIG. 3. Tube voltage dependent upon pressure for various arc currents.

[4] See reference 1, p. 43.

[5] Furnished by Baird Associates, Cambridge, Massachusetts.

intensity is approximately proportional to the current in the range measured. Figure 3 gives the tube voltage in dependence upon pressure for different currents. The tube voltage reaches a minimum at pressures between 2 and 3 mm Hg, the minimum voltage decreasing with increasing current. These data were obtained with alternating current as no direct current was available. It must be pointed out that, although the general shape of the curves will remain the same, the values for current, voltage, and ultraviolet output given will differ for different types of transformers and different ballast resistances in the circuit.

The authors are indebted to Dr. O. Oldenberg of Harvard University for many helpful discussions.

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 33, NUMBER 12 DECEMBER, 1943

A Note on Ray Tracing by Machine

JAMES H. ELLIS
Glendora, California

(Received October 7, 1943)

COMRIE'S method[1] of using a calculating machine to trace rays is convenient and time saving. This note describes a method of checking, also by machine, the correctness of the calculation. Following Comrie, the notation is that of Conrady.[2]

Comrie introduces three sets of constants, defined as:

$$A_1=1/r_1,\quad A_2=r_1/r_2,\quad \cdots,\quad A_n=r_{n-1}/r_n,$$

$$B_1=(L_1-r_1)/r_1,\quad B_2=(r_1-d'_1-r_2)/r_2,\quad \cdots,$$
$$B_n=(r_{n-1}-d'_{n-1}-r_n)/r_n,$$

$$C_1=N_1/N'_1,\quad C_2=N_2/N'_2,\quad \cdots,\quad C_n=N_n/N'_n;$$

and shows that the ray tracing can proceed in a compact tabular form by the use of the equations

$$\sin I_1=B_1\sin U_1,\quad \sin I'_1=C_1\sin I_1,$$
$$U_2=U_1+I_1-I'_1,$$

$$\sin I_2=A_2\sin I'_1+B_2\sin U_2,$$

$$\sin I'_2=C_2\sin I_2,\quad \cdots,\quad U_n=U_{n-1}+I_{n-1}-I'_{n-1},$$

$$\sin I_n=A_n\sin I'_{n-1}+B_n\sin U_n,$$

$$\sin I'_n=C_n\sin I_n,\quad U'_n=U_n+I_n-I'_n.$$

To derive the check, the well-known equation

$$(L-r)N\sin U=(L'-r)N'\sin U' \qquad (1)$$

is written, for the first surface, in the form

$$L'_1=r_1+(L_1-r_1)N_1\sin U_1/N'_1\sin U'_1$$
$$=r_1+B_1r_1N_1\sin U_1/N'_1\sin U'_1.$$

Then

$$L_2=L'_1-d'_1=r_1-d'_1+(L_1-r_1)N_1$$
$$\times\sin U_1/N'_1\sin U'_1.$$

Applying (1) to the second surface and substituting the above value for L_2, we have

$$L'_2=r_2+[r_1-d'_1+(L_1-r_1)N_1\sin U_1/$$
$$N'_1\sin U'_1-r_2]N_2\sin U_2/N'_2\sin U'_2.$$

Since $N'_1=N_2$ and $\sin U'_1=\sin U_2$, this can also be written

$$L'_2=r_2+(B_1r_1N_1\sin U_1+B_2r_2N_2\sin U_2)/$$
$$N'_2\sin U'_2.$$

The process may be repeated as often as desired, yielding the general formula

$$L'_n=r_n+S_n/N'_n\sin U'_n,$$

where

$$S_n=B_1r_1N_1\sin U_1+B_2r_2N_2\sin U_2+\cdots$$
$$+B_nr_nN_n\sin U_n.$$

Applying (1) to the expression for L'_n gives

$$L_n=r_n+S_n/N_n\sin U_n,$$

and since $\sin I_n=(L_n-r_n)\sin U_n/r_n$, with a similar expression for $\sin I'_n$,

$$\sin I_n=S_n/r_nN_n,\quad \text{and}\quad \sin I'_n=S_n/r_nN'_n.$$

These equations express data at any surface in terms of the constructional data of the system,

[1] L. J. Comrie, Proc. Phys. Soc. **52**, part 2, 246–52 (1940).
[2] A. E. Conrady, *Applied Optics and Optical Design* (Oxford University Press, 1929).

the data of the initial ray, and the angles which the intermediate rays make with the axis. If applied as a check on the correctness of the ray-tracing values, they will detect any error in the operations of deriving $\sin I'$ from $\sin I$, or $\sin I_n$ from $\sin I'_{n-1}$ and $\sin U_n$; but since they are derived without use of the equation $U+I=U'+I'$, they will not detect an error in the derivation of U' from U, I, and I', nor an error in converting sines to angles or the reverse.

To remedy this defect, the missing factor is introduced through use of the equation

$$\sin U'=N\sin U/N' + \sin(I-I')\sin(U+I)/\sin I \quad (2)$$

which contains only the conditions $U+I=U'+I'$ and $\sin I'=N\sin I/N'$, and simple trigonometric transformations.

Beginning with S_1, and employing (2),

$$S_1=B_1r_1N_1\sin U_1,$$

$$S_2=S_1+B_2r_2N_2\sin U_2=S_1+B_2r_2N_2\sin U'_1$$
$$=S_1+B_2r_2N_2(N_1\sin U_1/N'_1+F_1)$$
$$=S_1+B_2r_2(N_1\sin U_1+N_2F_1),$$

where $F=\sin(I-I')\sin(U+I)/\sin I$.

$$S_3=S_2+B_3r_3N_3\sin U_3$$
$$=S_2+B_3r_3N_3(N_2\sin U_2/N'_2+F_2)$$
$$=S_2+B_3r_3N_2\sin U_2+B_3r_3N_3F_2$$
$$=S_2+B_3r_3N_2(N_1\sin U_1/N'_1+F_1)+B_3r_3N_3F_2$$
$$=S_2+B_3r_3(N_1\sin U_1+N_2F_1+N_3F_2).$$

The general result is already evident; it is

$$S_n=S_{n-1}+B_nr_n(N_1\sin U_1+N_2F_1 + N_3F_2+\cdots+N_nF_{n-1}).$$

The use of this expression in computing S_n, instead of the one by which S_n was originally defined, ensures a complete check. It is simplest to check the values of $\sin I'_n$; agreement with the ray-tracing value guarantees the correctness of $\sin I'_n$ and the whole of the preceding calculation except the values for I_n and I'_n (since only F_{n-1} enters into the check, and not F_n). The only case in which the check gives a lessened accuracy is when I is small, but $U+I$ is large. This case may be met with an alternate expression for F, obtained by trigonometric transformation and expansion in series:

$$F=[1-N/N'+\tfrac{1}{2}\sin I' \times(\sin I-\sin I')+\cdots]\sin(U+I).$$

Further terms of the expansion are seldom necessary, and occasionally the term $1-N/N'$ suffices.

In using the check it is convenient to include in the original ray tracing the values of $\sin(U+I)$ and $(U+I)$, which Comrie omits. The calculation of N_nF_{n-1} is then a straightforward machine operation if tables for the difference between x and $\sin x$ are at hand, and the check computation may be written down in the form:

	$N_1\sin U_1$	(L_1-r_1)	S_1	$\sin I'_1$
N_2F_1	(a)	$(r_1-d'_1-r_2)$	S_2	$\sin I'_2$
N_3F_2	(b)	$(r_2-d'_2-r_3)$	S_3	$\sin I'_3$
N_4F_3	(c)	$(r_3-d'_3-r_4)$	S_4	$\sin I'_4$.

(a) is obtained by the addition of N_2F_1 to $N_1\sin U_1$, (b) by the addition of N_3F_2 to (a), and so on. S_1 is obtained as the product of (L_1-r_1) by $N_1\sin U_1$, S_2 by the addition to S_1 of the product of (a) and $(r_1-d'_1-r_2)$, S_3 by the addition to S_2 of the product of (b) and $(r_2-d'_2-r_3)$, and so on. The case of an object at infinity is met by writing 0 for $N_1\sin U_1$, and for S_1 the height at which the ray meets the first surface, multiplied by N_1.

Comrie's system breaks down at any plane surface not at the beginning of the system. The cure is to calculate L' for the last surface before the plane, using $L'=r(\sin I'+\sin U')/\sin U'$, then L for the plane by subtracting the separation, then L' for the plane, then L for the following surface, and to begin again with Comrie's method as though this were a first surface. The formula

$$L'=L[1+2N'\sin(U-U')/N\sin 2U]$$

allows the derivation of L' from L using only the sine table, at a plane surface. The check calculation likewise breaks down at a plane, and must be started over again.

Comrie gives expressions for checking the correctness of the computed constants A, B, C. It is perhaps as simple, when a paraxial ray forms part of the tracing, to trace that ray also by logarithms, agreement guaranteeing the correctness of the constants and checking the paraxial tracing.

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 33, NUMBER 12 DECEMBER, 1943

Sampling, Mixing, and Grinding Techniques in the Preparation of Samples for Quantitative Analysis by X-Ray Diffraction and Spectrographic Methods*[1]

JAMES W. BALLARD,[2] H. I. OSHRY,[3] AND H. H. SCHRENK[4]
Central Experiment Station, Bureau of Mines, Pittsburgh, Pennsylvania

A knowledge of the composition of atmospheric particulate matter is essential in the evaluation of health hazards from such contaminants. This may require the analysis of very small amounts of material collected from the air or the analysis of the source material of which large samples can be obtained. Owing to the small sample available and the complexities of these materials, ordinary methods of analysis are difficult and time-consuming. However, the x-ray and spectrographic techniques are particularly applicable to complex mixtures and require very small samples. In order to use these procedures it is necessary to have means of thoroughly mixing the very small sample with an internal standard, and also to have a sampling procedure which permits obtaining a very small portion which is representative of a large amount of material. This report describes methods which have been developed by the Bureau of Mines for obtaining small, representative samples and for mixing such samples intimately with an internal standard. The final mixing of the sample with the internal standard is carried out either in a small, specially designed ball mill or in a motor driven mortar and pestle. Precautions necessary in handling these small amounts of material are presented. Commonly used apparatus such as the Diamond mortar, ore grinder, and riffle are used in obtaining a representative sample. The method, while designed primarily to facilitate the evaluation of the health hazard associated with dusty atmospheres in the mineral industry, should have applications in other fields where the analysis of small amounts of material is necessary.

INTRODUCTION

PREPARATION of samples for analysis by x-ray diffraction methods[5,6] consists of obtaining a representative portion of each sample, mixing with an approximately equal quantity of internal standard, and grinding until the particles in the mixture are intimately mixed and sufficiently small to give satisfactory x-ray diffraction patterns.

Particle size is not as important for spectrographic analysis of powdered material as for x-ray diffraction analysis. The only restriction is that the particles be small enough so that a homogeneous mixture may be obtained for dilutions of one part of sample in several hundred parts of matrix or internal standard.

PRELIMINARY SAMPLING FOR X-RAY AND SPECTROGRAPHIC METHODS

A gross sample of ore or rock is first reduced to pieces less than $\frac{1}{4}$-inch mesh with a hammer and with a so-called "Diamond" mortar and pestle (*A*, Fig. 1). If the quantity of material is small, crushing can be completed in the mortar; otherwise, it is ground in the ore-grinding machine *B* so that the entire sample passes through a 60-mesh sieve.

The powder is mixed on a mixing cloth and then reduced in volume by passing it through a riffle (Fig. 2), which has 36 openings, each $\frac{1}{8}$ inch by 4 inches. Half of the sample is retained in each pan of the riffle. The half in one pan is set aside, and the riffling process is continued with the other half. This is continued until the sample has been reduced to about 3 grams. Portions of the 3 grams may then be prepared for either x-ray or spectrographic analysis.

The possibility of contamination of the sample by the plates of the ore grinder was checked by analyzing a piece of granite, part of which was ground in the ore grinder and part of which was

---

* Paper presented at the Twenty-Eighth Annual Meeting of the Optical Society of America, Pittsburgh, Pennsylvania, October 7–9, 1943.

[1] Published by permission of the Director, Bureau of Mines, U. S. Department of the Interior, Washington, D. C.

[2] Former associate physicist, Gas and Dust Section, Central Experiment Station, Bureau of Mines, Pittsburgh, Pennsylvania; present address: Mine Safety Appliances Company, Pittsburgh, Pennsylvania.

[3] Former assistant physicist, Gas and Dust Section, Central Experiment Station, Bureau of Mines, Pittsburgh, Pennsylvania; present address: Koppers United Company, Butadiene Division, Monaca, Pennsylvania.

[4] Chief chemist, Health Division, Bureau of Mines.

[5] James W. Ballard, H. I. Oshry, and H. H. Schrenk, "Quantitative analysis by x-ray diffraction. 1. Determination of quartz," Bureau of Mines Report of Investigations 3520 (1940), 10 pp.

[6] Since the x-ray sampling technique to be described is used in part for determining quartz (free silica) using fluorite as an internal standard, quartz and fluorite will be used in the description of the sampling, mixing, and grinding technique.

FIG. 1. Apparatus for preparing samples: *A*, "Diamond" mortar and pestle; *B*, ore grinder; *C*, ball mills for x-ray diffraction; *D*, *E*, ball-mill clamp and wrench; *F*, motorized rollers; *G*, heating coils; *H*, electrical ground; *I*, ball mills for spectrographic analysis.

crushed in the "Diamond" mortar. The quantity of iron found in each instance was very small, but the "Diamond" mortar and pestle method of crushing samples was found to produce less contamination than any other method available in the laboratory.

PREPARATION OF SAMPLES FOR X-RAY ANALYSIS

A portion of the 3 grams remaining from the last riffling operation is crushed in a "Diamond" mortar to pass through a 200-mesh sieve for x-ray analysis. This powder is mixed on glazed paper and quartered. The sample thus obtained is suitable for mixing with an internal standard and grinding to the size range required for the diffraction patterns. The final mixing and grinding may be done either with a ball mill or with a motor-driven mortar and pestle.

Ball Mill Technique

The ball mills *C* (Fig. 1) are made of stainless steel tubing $2\frac{1}{2}$ inches inside diameter and are $2\frac{1}{4}$ inches deep. The stainless steel ends (one has a $\frac{1}{8}$-inch hole at the axis) of the mills are screwed on the cylinders against lead gaskets, which are trimmed flush inside the mills so as not to grind off and contaminate the sample. The ends are put on with the help of the vise and wrench, *D* and *E*. Usually 1 gram of fluorite, the internal standard, is mixed with 2 grams of the sample, although smaller or larger quantities of material may be satisfactorily mixed and ground. The powder mixture is added to a mill with about 210

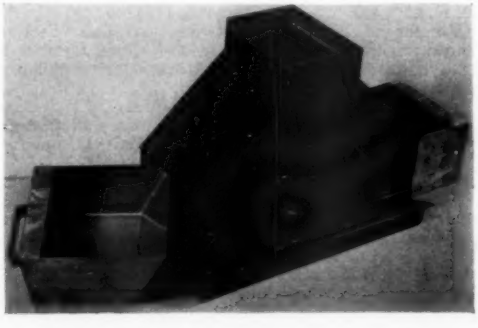


FIG. 2. Riffle for obtaining representative portion of sample.

"Grade A" chrome-steel balls ¼ inch in diameter, and enough dispersing liquid (methanol) is added to cover the balls. This fills the mill to about one-third its depth and prevents spilling of the sample through the vent in the end of the mill when it is placed on rollers for mixing and grinding. Rotation at 22 r.p.m. is accomplished on parallel steel shafts *F*, the separation of which may be changed to accommodate different diameter mills and mixing cylinders. The mills are heated during the last hour of rotation by four rows of parallel heating coils *G* to evaporate the dispersing liquid while the mills are rotating. The rollers and mills are connected electrically to the ground through wire *H*.

Dispersing Agent

The results were not consistent when the mills were first used for dry-grinding a sample containing quartz, calcite, and fluorite; part of the sample collected on the balls and the mill. X-ray analysis showed that more quartz was present in the loose material than in the collected material. When samples that were good electrical insulators were ground, it was found with an electroscope that a considerable electrostatic charge was developed and the mills became charged.

The samples in the mills were then dispersed in petroleum ether; but the same difficulties were experienced, although to a lesser extent. Studies of the electrical resistance of several liquids led to adoption of methanol as a dispersing agent for the samples in the mills. Methanol was chosen because of its electrical characteristic, because of its low boiling point, and because a large number of minerals are insoluble in it. The use of a liquid dispersing agent in the mills made it necessary to prevent selective settling of the sample at the end of the mixing period in the mills. This was accomplished by evaporating the liquid while the sample was mixing.

Grinding Efficiency

To test the grinding efficiency of the mills, samples were prepared containing 6 grams of quartz powder sifted through a 200- onto a 325-mesh sieve and 1 gram of internal standard, fluorite, sifted through a 200-mesh sieve. These samples were put in the mills and rotated for various time periods up to 46 hours. Diffraction patterns were made, and the ratio of the intensity of the 3.35A quartz line to the 3.16A fluorite line was determined. Figure 3 shows the variation of this ratio with the time of grinding. It may be seen from the figure that the ratio of line intensities becomes independent of the time after about 24 hours grinding. Since many of the samples received are smaller in particle size and softer than the coarse quartz used in the test, 24 hours was chosen and found satisfactory as the time of grinding for all samples prepared in the ball mills. Figure 4 shows x-ray diffraction patterns of a sample in three stages of the grinding process. Satisfactory diffraction patterns are obtained when the particles in the samples are smaller than 10 microns but not small enough to cause appreciable broadening of the lines on the x-ray film. Suitable densitometric technique aids in decreasing the effect of particle size on the analysis.

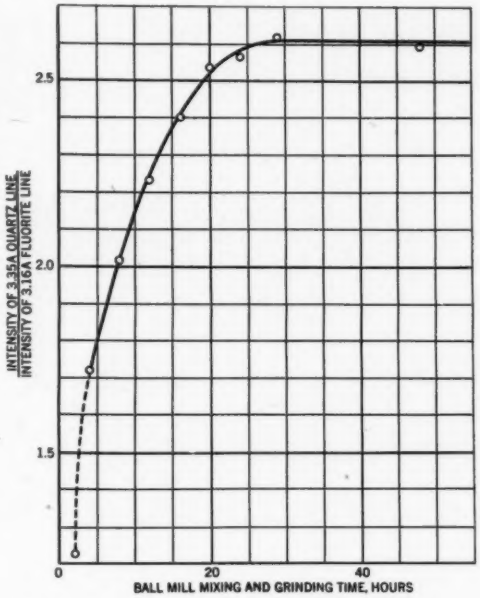


FIG. 3. The effect of grinding and mixing time upon the ration of the intensity of the quartz line to the intensity of the fluorite line for samples of 2 grams of −200–325-mesh quartz mixed with 1 gram of −200-mesh fluorite.

Mixing Efficiency

The mixing efficiency of the ball-mill technique was tested by comparing the precision obtained

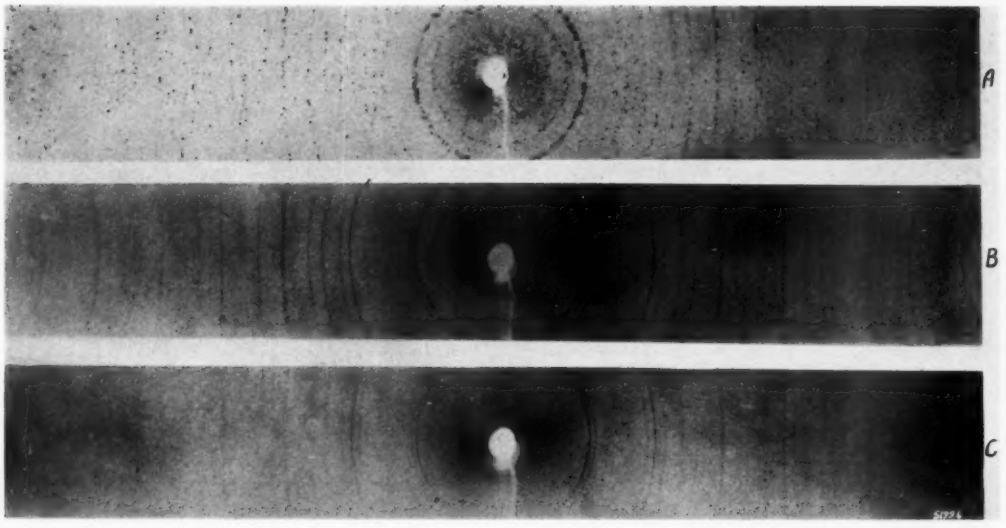


FIG. 4. Effect of grinding (particle size) on x-ray diffraction pattern: *A*, sample; *B*, 2 hours grinding; *C*, 24 hours grinding.

by taking several pictures of a single portion of a sample with the precision obtained with pictures taken of several portions from the same sample. The first set of pictures, indicating the precision of the x-ray diffraction technique, gave an average deviation of 1.2 percent from the mean value of the intensity ratio. The average deviation from the mean intensity ratio was 3.8 percent for the pictures taken of different portions of the sample. As a further check, measurements were made using different portions of each of several other samples. The results of all these samples indicates that the error due to mixing is of the order of 3 percent. The above data were obtained by making seven measurements of the intensity ratio on each film to improve the precision of the densitometric work. The mixing error, which includes handling the samples and loading the wedge for exposure to the x-ray beam, may be reduced by exposing more sample to the x-ray beam and by averaging the results obtained from more than one portion of the sample. The former was accomplished by using a lead slit collimating system for the x-ray beam along with a wide metal wedge for holding the sample.[7]

[7] See footnote 5, page 1.

Contamination

When samples are mixed and ground in mills, there is a certain amount of contamination due to material ground from the mills and balls (Fig. 5). Lines from this metallic contamination do not appear on the x-ray film for the times of exposure usually used for quantitative analysis when

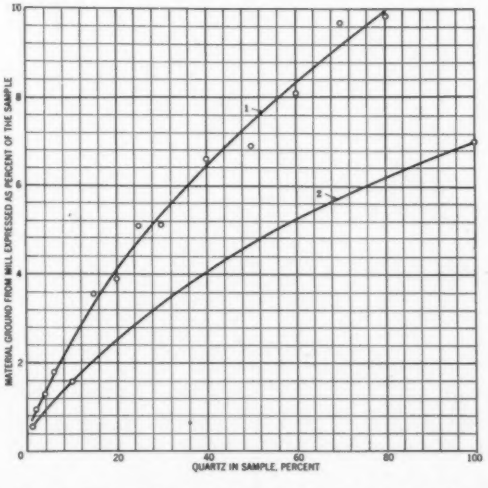


FIG. 5. Material ground from ball mill as a function of the percent quartz present in the sample. 1, stainless steel mill; 2, hardened tool-steel mill.

x-rays from a copper target are used. For long exposures, a few strong lines may appear on the film. Because of their small number and their position on the film, these lines do not interfere with most analyses. However, for the determination of some chemical compounds, the interfering lines must be considered. The metallic contamination decreases the amount of radiation diffracted from the sample but does not extend the time of exposure appreciably. The addition of the metal to the sample does not affect the quantitative determinations since the ratio of weights of the internal standard and the chemical compound to be determined remains the same.

Cleaning

Since the x-ray technique is not sensitive to traces of materials, the mills may be cleaned with gritty hand-cleaning preparations such as those used by mechanics, using very hot tap water, a stick, and a cloth. After thorough cleaning and rinsing in hot water, the mills are dried with a clean towel. The balls are cleaned by putting them into a metal container with some of the cleaning preparation and stirring with a cloth on the end of a stick. They are rinsed several times in very hot tap water, drained on a stainless steel screen, and spread on a clean cloth where they are dried by rolling them with a clean cloth on the hand. No difficulties were experienced with rusting, and no further cleaning of the mills or balls was found necessary for x-ray work.

Mortar and Pestle Technique

Although 24 hours for mixing and grinding is not excessive when other work may be accomplished during this period, it is occasionally necessary to make an analysis in the shortest possible time. Therefore, a study was made with a motor-driven mortar and pestle as a possible means of decreasing the time of mixing and grinding samples. An agate mortar and pestle could not be used for determining quartz because agate gave a diffraction pattern whose lines interfered with those of quartz. A hardened tool-steel mortar (8.7 cm in diameter) and pestle (Fig. 6) was chosen because the contamination did not affect the results of analysis for quartz, and a large number of other chemical compounds. Also, less metallic contamination was introduced than from the ball mill and therefore greater diffraction efficiency of a sample for copper x-rays. The sample is usually mixed and ground dry until adherence to the metal surface occurs. Methanol is then added in sufficient quantity to disseminate the material. Grinding is terminated when most of the methanol has evaporated, leaving a thick paste, which is allowed to dry. If the particles are too large at the end of a first

FIG. 6. Motor-driven mortar and pestle used for mixing and grinding sample.

evaporation, more methanol may be added and the process repeated. Usually three applications of methanol are sufficient. The material is transferred to glazed paper with a metal spatula and a camel-hair brush, mixed, and a portion for analysis obtained by quartering.

Grinding Efficiency

The grinding efficiency of the mortar and pestle was tested with a sample of quartz powder sifted through a 200- onto a 325-mesh sieve. A microscope was used to study the progress of the grinding of the material. Two hours were required to grind the quartz, so that there were no particles larger than 10 microns. Extended periods of grinding reduced the particles to sizes so small that the lines on the x-ray film broadened appreciably. This decreased the accuracy of quantitative analysis, and grinding should therefore be stopped before the particles are reduced too greatly in size. In general, the weight and hardness of the sample determine the time of

grinding. For a sample of quartz and fluorite weighing 0.750 gram and sifted through a 200-mesh sieve, about 2 hours is required.

Samples ground dry in the metal mortar and pestle frequently did not mix and grind satisfactorily because of differential disintegration and a tendency of fine particles to adhere to the metal surfaces. To grind the harder components further and mix the sample further without excessive reduction of the particle size of the more

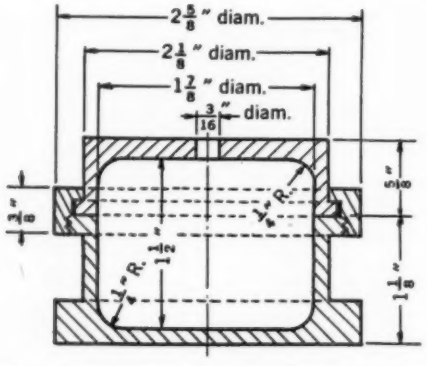


FIG. 7. Section of nickel mill used to mix sample.

friable components of the sample, methanol was added when adhesion to the mortar started.

Mixing Efficiency

The mixing efficiency of the mortar and pestle technique was tested by taking diffraction patterns of five portions from each of six samples, all containing 10 percent quartz. The intensity ratio between the quartz line and the fluorite line was measured once on each pattern. The average deviation from the mean value of the intensity ratio for the six samples was found to be 3.1 percent. A similar test on a sample containing 30 percent quartz gave an average deviation of 4.7 percent from the mean. The results indicate that mixing in the mortar is as uniform as in the ball mills; however, more attention is required.

It was found that samples containing magnetic material would be partly separated by attraction to the mortar. This does not, however, affect the results of analysis for the non-magnetic constituents of the samples, since the ratios of weights of the internal standard and the non-magnetic constituents are not affected by extracting the magnetic material.

Technique for Special Samples

For very small samples the above technique must be modified. For samples weighing a few milligrams up to several hundred milligrams, such as those collected from the air with an electrostatic precipitator,[8] it is convenient to use the mortar technique described above but with even greater precaution not to grind the particles too fine. A well-polished tool-steel mortar and pestle, with hand grinding, is used for samples less than 1 milligram, such as those collected from the air with the midget impinger.[9] The sample is transferred directly into the mortar and mixed with a weighed amount of some material not affecting the analysis, such as soluble starch, and with a weighed amount of internal standard. These small quantities may be weighed very rapidly and accurately with a torsion balance having three ranges, 0 to 25, 25 to 50, and 50 to 75 milligrams. After mixing and grinding the total sample is weighed, and the weight of the original material is determined by difference. The sample may be mixed further before analysis with a spatula on glazed paper or on a plate of glass.

For very complex samples, methods of separating the samples into less-complex portions sometimes aids in making an analysis. For samples containing magnetic material, magnetic separation simplifies the work of analysis and gives satisfactory results with x-rays from a copper target, eliminating the necessity of changing to more penetrating radiation such as that from a molybdenum target. Other samples may be separated into their various constituents by flotation or settling. Weight separations sometimes may be made by placing a prepared powder in a liquid in a large watch glass and moving it with a rocking motion. For some samples, powder may be separated into its components by the use of heavy liquids.[10]

[8] E. C. Barnes and G. W. Penney, J. Ind. Hyg. Tox. 20, 259–265 (1938).

[9] Carlton E. Brown and H. H. Schrenk, "A technique for the use of the impinger method," Bur. Mines Inf. Circ. 7026 (1938), 20 pp.

[10] R. G. O'Meara and J. Bruce Clemmer, "Methods of preparing and cleaning some common heavy liquids used in ore testing," Bur. Mines Rept. Invest. 2897 (1928), 6 pp.

PREPARATION OF SAMPLES FOR SPECTROGRAPHIC ANALYSIS

The spectrographic technique employed for the quantitative determination of the major constituents of mineral powder[11] involves mixing a portion of the sample weighing less than 12 milligrams with 1 gram of nickel oxide to act as internal standard and spectroscopic buffer, and one-half gram of pure graphite powder to serve as stabilizer in the arc. About 10 milligrams of this mixture are burned in the arc to make the spectrogram. Of the portion burned in the arc, about 0.08 milligram is the original sample.

For the proper preparation of the sample, it has been found that the nickel oxide, the graphite, and the mineral powder must be small enough to pass through a 400-mesh sieve. This is especially necessary for the mineral, since only 0.08 milligram of the sample is burned. This reduction in particle size insures the presence of at least several thousand sample particles in the 0.08-milligram portion and decreases the probability that the small amount used differs significantly in composition from the entire sample.

The study of sample mixing and grinding for x-ray method indicated that the most satisfactory method for mixing the spectrographic samples would be the use of ball mills. The sensitivity of the spectrographic technique to small amounts of impurities required a procedure that would reduce contamination of the samples by the mills to a minimum during the mixing process. This was accomplished by using mills made from nickel (Fig. 7). The contamination added to the sample during the mixing process is therefore principally nickel which, added to the gram of nickel oxide already in the sample, has a minimum effect on the results of an analysis. The minor impurities in the nickel are seldom significant.

The mill shown in Fig. 7 was designed with rounded corners to facilitate cleaning and also could be made liquid-tight without the use of gaskets. The mills were machined from nickel castings, and the flanges of the lid and body of each mill were ground together. When a thin film of petrolatum was placed on the flange of the bottom part of the mill pressure of the ring which held the mill together made a liquid-tight joint between the two sections. The top section of the mill is provided with a $\frac{3}{16}$-inch hole at the axis so that the liquid dispersing agent may evaporate during the mixing process.

The procedure used for preparing the sample for spectrochemical analysis is as follows: The material from the final riffling operation is crushed in the "Diamond" mortar until it passes through a 400-mesh screen. It is then mixed on glazed paper and quartered. The quartered sample is placed in one of the mills, along with about 70 grams of nickel shot between $\frac{1}{8}$ and $\frac{3}{16}$ inch in diameter and 10 milliliters of reagent-grade methanol as dispersing agent. The mill is sealed with petrolatum and placed on rollers,

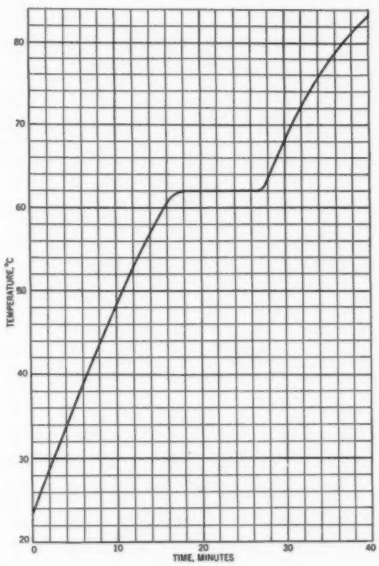


FIG. 8. Temperature of mill as a function of time after heaters are turned on.

and the heating coil is turned on. The sample is allowed to rotate until the methanol is completely evaporated. Figure 8 shows a curve of the temperature of the mill against time of rotation, which indicates the time for complete evaporation. Mills were opened at various intervals of times, and the condition of the sample was observed. It was found that at the end of 35 to 40 minutes the exterior of the mill reached 75°C, and the sample was completely dry. In subse-

[11] H. I. Oshry, J. W. Ballard, and H. H. Schrenk, J. Opt. Soc. Am. **32**, 672–680 (1942).

quent samples, when the mill reached this temperature it was removed from the rollers and allowed to cool. When cooled, it was opened and the sample removed with the aid of a rubber policeman. The sample was separated from the nickel shot by screening through a 12-mesh stainless steel screen. This procedure adequately mixed the sample so that a small portion may be taken for mixing with the nickel oxide and graphite powder.

For preparing the final mixture, about 70 grams of the nickel shot are added to a clean mill. The nickel oxide and graphite powder are weighed and transferred to the mill along with a portion of the mineral sample that has been weighed on a torsion balance. Ten milliliters of reagent-grade methanol are added, and the mill is sealed and placed on the rollers until the methanol has completely evaporated and the temperature reaches 75°C, which requires 35 to 40 minutes. The sample is cooled as before and stored in a small vial. That 35 to 40 minutes is adequate for satisfactory mixing was checked by mixing samples for periods up to 8 hours. No increase in the homogeneity of the sample was found with longer times of mixing, but the problem of contamination became increasingly great.

The problem of testing the homogeneity of mixing is more complex for the spectrographic technique than for the x-ray technique, since the sample is destroyed in preparing the spectrogram and the sampling error cannot be separated from the errors in the arc and in the photometric technique. The testing, therefore, was done on a relative basis, and the mixing was considered adequate when the difference in the precision of measurements between successive spectra approached the same order of magnitude as the precision obtained in densitometric work. The average deviation from the mean intensity ratio between a line due to an element to be determined in the sample and the line due to the nickel was found to be of the order of 5 percent for five spectra made from portions of the same sample mixture.

The total contamination with materials other than nickel introduced into the sample by material ground off the mill and the nickel shot during the mixing process is small and approximately the same for all samples. It has been found that the extent of contamination may be determined by mixing just the nickel oxide and the graphite powder and analyzing it for the contaminants. The amounts found by this determination may then be used to correct the results of all samples prepared with that supply of nickel oxide and graphite.

Cleaning of the mills and the nickel shot is very important, since very small amounts of contamination may introduce appreciable errors in the analytical results. The mills are first cleaned with a cloth moistened with cleaners' naphtha. They are then cleaned with Bon Ami while rotating rapidly in a lathe. After being rinsed in hot tap water, they are dried with a clean towel, rubbed with a clean cloth moistened with xylol, washed in hot soapy water, rinsed again in hot tap water, and dried. The nickel shot is cleaned by repeated washing in hot, soapy water followed by cleaning with cleaners' naphtha. This cycle is repeated until no turbulence is seen after the shot have been poured into fresh cleaners' naphtha and stirred. The shot are then rinsed in very hot water, drained on a stainless steel screen, and dried by spreading on a clean towel. If this procedure is followed, contamination is reduced to a minimum.

DISCUSSION

The ball-mill technique for x-ray analysis has the advantages of great homogeneity of mixing and of reduction of the particles to a size range that gives good diffraction patterns for densitometric measurements. The disadvantages are the time required for grinding and mixing and for cleaning the mills and balls. The mortar and pestle grind the sample in less than 2 hours and are easily cleaned. Less iron is introduced into the sample by the mortar and pestle technique than by the ball-mill technique, making the diffraction patterns more intense for a given time of exposure when copper radiation is used. However, the sample must be watched when mixed in the mortar. It sometimes "crawls" on the walls of the mortar and up the sides of the pestle and must be cleaned several times during mixing to prevent selective separation of the sample. This separation may be minimized by mixing the sample dry until the particles begin

to "rub out" on the mortar, and then adding methanol. If the sample is ground and mixed too long in the mortar and pestle, the particles will become too small for accurate quantitative analysis, because of the broadening of the lines of the x-ray diffraction pattern.

The ball-mill technique for sample mixing and grinding is recommended for best accuracy. However, where time is a factor and the best accuracy is not required the mortar technique is satisfactory, especially if the average result of analyses of more than one portion of the sample is taken.

The results obtained with the mixing of samples for spectrochemical analysis indicate that the time required for mixing is comparatively short if the particles are reduced sufficiently in size before they are mixed. Careful and complete cleaning of the mills and the shot is necessary to prevent errors due to contamination. The problem of contamination is reduced further by making the mills of the same material as the matrix in which the sample is mixed.

SUMMARY

Methods of sampling for quantitative analysis by x-ray diffraction and spectrographic methods are described. For gross material a "Diamond" mortar and pestle, ore grinder, mixing cloths, and riffle are used for obtaining a small representative sample. For x-ray diffraction analysis a small portion of this material is mixed with an internal standard and reduced to homogeneous mixtures of satisfactory particle-size range in ball mills or in a motor-driven mortar and pestle. A ball-mill technique for mixing samples for spectrographic analysis is described which minimizes the effects of sample inhomogeneity on quantitative analyses. The methods, although applied to specific problems, may be of use in other techniques where a very few milligrams of material must be made to represent a large gross sample.

The Graphical Representation of Small Color Differences†

D. L. MacAdam
Eastman Kodak Company, Rochester, New York
(Received October 22, 1943)

IN a note bearing this title,[1] W. D. Wright directs attention to personal variations of considerable magnitude which are indicated by existing data[2,3] on the noticeability of small color differences and suggests that some plane projective transformation of the I.C.I.* chromaticity diagram may prove adequate as a uniform chromaticity chart. Wright illustrates this thesis with a representation of his experimental data in the R.U.C.S. diagram advocated by Breckenridge and Schaub.[4]

Likewise, Fig. 1 shows the twenty-five ellipses derived from the observations of P.G.N.[3] transformed to a diagram that is metrically equivalent to that of Breckenridge and Schaub. The spacing of the data for Nutting is at least as uniform in Fig. 1 as Wright's data shown in his Fig. 4. Such a diagram may therefore, as Wright suggests, serve many useful purposes. It seems reasonable to suggest further that in order to merit consideration, even as a practical approximation, every proposed representation of the noticeability of color differences should fit the experimental data at least as well as the uniform chromaticity scale diagram published by Judd,[5] from which Fig. 1 and Wright's Fig. 4 were derived.

As an example of the application of this criterion, Figs. 2 and 3 show that the experimental data for P.G.N. are represented less uniformly (i.e., by ellipses of greater eccentricity)

† Communication No. 947 from the Kodak Research Laboratories.

[1] W. D. Wright, J. Opt. Soc. Am. **33**, 632 (1943).

[2] W. D. Wright, Proc. Phys. Soc. (London) **53**, 93 (1941).

[3] D. L. MacAdam, J. Opt. Soc. Am. **32**, 247 (1942).

* Initials of the International Commission on Illumination. Wright uses the abbreviation C.I.E., which are the initials of the official French title: Commission Internationale de l'Eclairage.

[4] F. C. Breckenridge and W. R. Schaub, J. Opt. Soc. Am. **29**, 370 (1939).

[5] D. B. Judd, J. Opt. Soc. Am. **25**, 24 (1935).

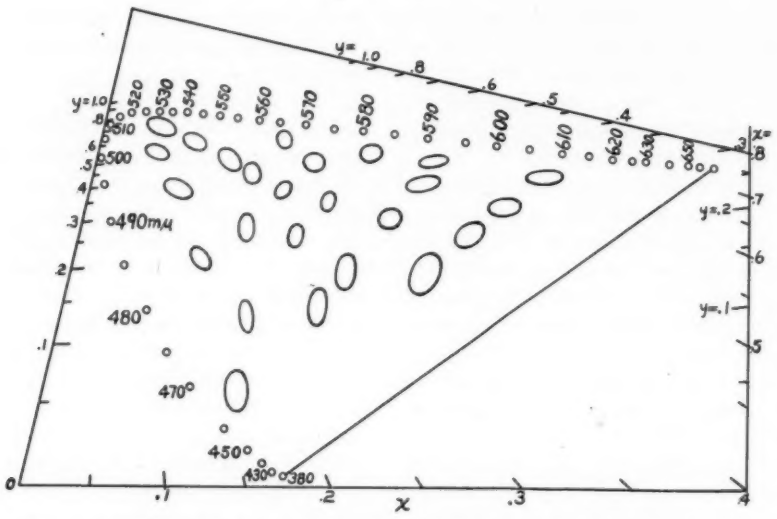


FIG. 1. Magnified (10×) loci representing standard deviations of color matching (by P.G.N., reference 3) in various directions for various locations of Judd's U.C.S. diagram (reference 5).

in the diagram recommended by Moon and Spencer[6] than in Judd's more convenient projective transformation, as illustrated in Fig. 1. Attention is called to the fact that the deviations of experimental points from the ellipses in the I.C.I. diagram as originally reported rarely exceed 20 percent.

If convenience were the only consideration, there would be no reasonable objection to Wright's proposal, but so many conclusions of practical as well as theoretical import seem derivable from the geometric properties of a truly uniform chromaticity diagram that discontinuance of the search for an adequate representation or general acceptance of any fundamentally inadequate diagram would be most unfortunate. Although personal variations of considerable magnitude are indicated by Wright's Fig. 3, the curves for all of the observers exhibit striking agreement concerning the existence and even the location of a minimum in the realm of real colors. The writer has published two proofs[7, 8] that no plane projective transformation can possibly provide uniform spacings for data which, when plotted as in Wright's Fig. 3, are represented by curves having such minima. All plane projective transformations in effect approximate such a representation of the data either by a horizontal straight line or by a portion of a parabola, the vertex of which is tangent to the horizontal axis at some value of l beyond the realm of real colors. Since practically all the data originally reported in this form by Wright[2] and by the writer exhibit such minima, the conclusion seems inescapable that projective transformations are fundamentally inadequate for uniform representation of the experimental facts.

[6] P. Moon and D. E. Spencer, J. Opt. Soc. Am. **33**, 260 (1943).

[7] D. L. MacAdam, J. Opt. Soc. Am. **32**, 2 (1942).

[8] D. L. MacAdam, J. Opt. Soc. Am. **33**, 18 (1943). Mention should have been made of the following exceptions to the statement that Figs. 1, 2, and 3 of this reference represent the data within 30 percent. The value of g_{11} computed from Fig. 47 of reference 3 is 51, but the value interpolated from Fig. 1 of reference 8 is 85, an error of 67 percent. The value of g_{22} computed from Fig. 36 of reference 3 is 14.4, the value 24 indicated in Fig. 3 of reference 8 being 67 percent too great. Percentage error is an inadequate specification of the accuracy of Fig. 2 of reference 8 since small values may have great percentage errors without serious consequences. The accuracy of this figure may be described more significantly by comparison of the inclination of the major axis of each observed ellipse with that computed from the values indicated by Figs. 1, 2, and 3 of reference 8. The inclinations thus computed are within five degrees of those indicated by nineteen of the twenty-five ellipses reported in reference 3. Five others are within ten degrees of those indicated in reference 3. The major axis of the ellipse shown in Fig. 23 of reference 3 is inclined at 62 degrees above the horizontal. The inclination computed from Eq. (4a) and Figs. 1, 2, and 3 of reference 8 is 73.2 degrees, an error of 11.2 degrees.

The writer does not feel qualified to outline the conclusions that proponents of theories of color vision can derive from uniform chromaticity diagrams, but he is so well aware of potential developments along these lines that he cannot concur in the recommendation of a type of representation which is clearly and fundamentally inadequate. Several implications of more direct practical importance can be cited to the same effect.

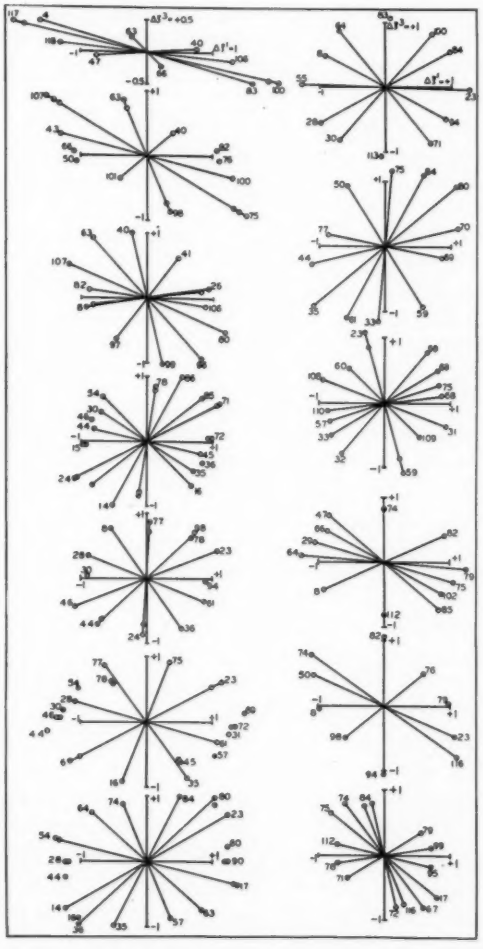


FIG. 2. Distributions of experimental points representing standard deviations of color matching (by P.G.N.) as they appear in the (ξ^1, ξ^3) diagram of Moon and Spencer (reference 6). Figures 23 to 29 in reference 3 correspond to the distributions shown in sequence from top to bottom of the left column. Figures 30 to 35 in reference 3 correspond to the distributions shown in sequence from top to bottom of the right column.

Schrödinger[9] suggested that the geodesic (i.e., the series of colors involving the least number of just noticeable steps) from the neutral (or white) to any spectrum color might be identical with the series of colors of increasing saturation but constant (subjective) hue. Geodesics cannot possibly be determined if the search for a uniform

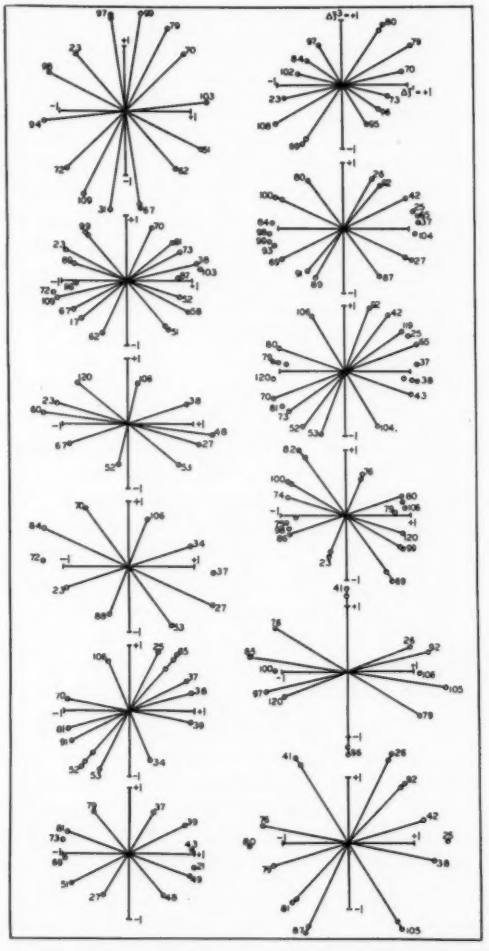


FIG. 3. Standard deviations of color matching (P.G.N.) as they appear in the (ξ^1, ξ^3) diagram of Moon and Spencer. Figures 36 to 41 in reference 3 correspond to the distributions shown in sequence from top to bottom of left column. Figures 42 to 47 in reference 3 correspond to distributions shown in sequence from top to bottom of right column. The distributions shown above corresponding to Figs. 31 and 42 of reference 3 contain errors not influencing the general conclusions. Figure 7 of reference 6 shows correctly the distribution for the first of these.

[9] E. Schrödinger, Ann. d. Physik **63**, 397, 427, 481 (1920).

TABLE I.

Nearest color temperature Degrees K	Estimated conjunctive wave-length (millimicrons) Davis	Judd	DLM
2000	584.3	585.6	586.2
2500	579.8	580.5	580.3
3000	575.8	575.9	575.2
3500	573.3	571.6	571.2
4000	570.8	568.0	567.5
5000	567.7	560.6	562.1
6000	565.4	553.3	559.0
6500	564.7	549.2	557.0
7500	563.3	542.4	553.5
10,000	561.1	527.2	547.0

chromaticity diagram is halted by acceptance of a plane projective diagram. Schrödinger's suggestion is at least entitled to a test and the development of an accurately uniform chromaticity representation for at least one observer is necessary for the determination of geodesics which may be compared with series of colors of constant hue.

Loci of equal correlated color temperature have been proposed by Davis[10] and by Judd.[11] Such loci should be derivable from experimental data on equally noticeable color differences. These loci are conveniently specified by the wave-length corresponding to the intersection of the spectrum locus with straight extensions of the iso-color-temperature loci. This was named "conjunctive wave-length" by Davis.[10] Column 1 of Table I lists selected color temperatures. Column 2 gives the conjunctive wave-lengths for the iso-color-temperature loci proposed by Davis. Column 3 gives the conjunctive wave-lengths of the iso-color-temperature loci defined by Judd as the perpendiculars to the locus of Planckian colors in the metrically equivalent prototype[5] of Fig. 1 and of the R.U.C.S. diagram of Breckenridge and Schaub advocated by Wright. Column 4 of Table I lists the conjunctive wave-lengths of loci of iso-color-temperature derived from the observations of P.G.N. by the use of the method outlined by the writer. These results agree well with those of Judd and Davis from 2000°K to 3000°K; they agree with Judd's up to 4000°K, and fall about halfway between those of Judd and Davis from 5000°K to 10,000°K. The differences from the values derived from Judd's work indicate significant differences between conclusions derived from (1) the most accurate representation of experimental data which the writer is able to suggest and (2) a plane projective transformation metrically equivalent to that recommended by Wright.

The determination of correlated color temperature is but one of a general class[8] of similar problems which can be solved by use of an adequately uniform representation of chromaticity differences. The use of a plane projective transformation for such purposes would utilize available data to considerably less than the fullest advantage. In reference 8 the writer has tried to show that accurate utilization of experimental data need not be less convenient than the use of a plane projective transformation of the I.C.I. diagram. He has acknowledged the desirability of much more extensive data and does not urge final acceptance of his representation of his own data. In his opinion, however, no additional data are likely to become available for the duration of the war and probably for some time thereafter and existing data indicate that representations similar to the one he has published[8] will be necessary if the full implications of experimental data are to be recognized.

It should not be surprising that the data for P.G.N. alone, of the six observers cited by Wright, have been used for the derivation of non-linear transformations of the I.C.I. diagram. The data for P.G.N. are the only data in a form convenient for analysis by the methods of differential geometry. Silberstein[12] indicated in 1938 that the loci of colors equally noticeably different from each of many fixed colors would be the most useful data for the derivation of a color metric. The writer has tried to obtain such loci from Wright's data by displacing slightly, parallel to themselves, the line segments in Wright's Fig. 2 so as to pass through common midpoints. The scatter of the ends of these segments about the most reasonable ellipses drawn through them was much greater than in Figs. 23 to 47 published by the writer.[3] Discrepancies of similar magnitude have also been noticed when the results for P.G.N. represented in Figs. 8 to 19 were compared with those

[10] R. Davis, Bur. Stand. J. Research **7**, 659 (1931).
[11] D. B. Judd, J. Opt. Soc. Am. **26**, 421 (1936).
[12] L. Silberstein, J. Opt. Soc. Am. **28**, 63 (1938).

represented in Figs. 23 to 47. These discrepancies are attributed to long-term changes in the standard deviations for P.G.N. and similar changes in criterion may be responsible for the failure of Wright's data to define satisfactory loci of equal noticeability around fixed central colors.

These remarks are not intended as any criticism of Wright's splendid research but merely to indicate why Wright's data were not employed in the development of a color metric. The data of D.L.M. were not used for the same reason. The writer feels that there is no disagreement as to experimental facts between his work and Wright's and that only a difference of opinion is involved, concerning the advisability of overlooking certain of those facts in the interest of immediate convenience.

Letter to the Editor

A Note on Theoretical Color-Blindness Frequencies

JOHN R. PLATT
Northwestern University, Evanston, Illinois
October 10, 1943

THE actual distribution of color-blindness in the population, as given by Judd,[1] can be compared with the frequency ratios predicted for sex-linked recessive genes by simple genetic theory.

If a fraction a of all X chromosomes bears a defective gene of a certain kind (color-blindness), then the chance of a male (one X chromosome) having the corresponding phenotype CB (color-blind) must be also a. The probability of a female (two X chromosomes) having one defective and one normal chromosome and therefore being Car (carrier) is evidently twice the product of the odds on each event separately, or $2a(1-a)$. Similarly, the odds that a female has both chromosomes defective is a^2; only in this case is the female expected to have the phenotype CB.

If several types of defect, CB_a, CB_b, CB_c, etc., occur in the male with frequency a, b, c, etc., the probability of corresponding female carriers, Car_a, etc., becomes

$$2a(1-a-b-c-\cdots), \text{ etc.},$$

and the total number of carriers is

$$2(a+b+c+\cdots)(1-a-b-c-\cdots).$$

The odds on female color-blindness become a^2 for CB_a, b^2 for CB_b, c^2 for CB_c, etc. The odds that a woman has two genes with different defects are $2ab$ for CB_{ab}, $2ac$ for CB_{ac}, $2bc$ for CB_{bc}, etc., making the total fraction of CB women equal to $(a+b+c+\cdots)^2$.

These predictions agree exactly with those given by Waaler[2] but the present method of approach and the computational algebra seem to be simpler than his. Waaler's presentation also suffers from several errata, especially on p. 287 where the vital term xv has been omitted from the equation of y. (This error has not been carried through the computations, however, and the final value of y, $y=2x-2z$, is correctly obtained.)

This theory makes it possible to be much more definite about the number of female carriers than is indicated by the statement in Judd's article ". . . at least 8 percent of women must be transmitters. . . ." If Judd's figure of 8 percent for the fraction of CB males is put in the above equations, the number of Car females comes out 14.72 percent. To these Car females with one defective gene, Waaler would also add such cases as that of a woman with two different defective genes, either of which can produce defective vision in her children, but both of which together cancel in her to produce quasi-normal vision. By his theory, 0.24 percent of women are of this latter type; so the total fraction of women having normal visual systems themselves but capable of transmitting color-blindness becomes 14.96 percent. It is startling to realize that one woman in seven carries this defect.

It is gratifying to see that when account is taken of Waaler's findings on the relative recession, dominance, and cooperation of the different CB genes, then these predictions of genetic theory are in excellent agreement with the observed percentage relations of the four defective visual systems in men and women as quoted in Judd's article.

[1] Deane B. Judd, J. Opt. Soc. Am. **33**, 294 (1943).
[2] G. H. M. Waaler, Zeits. f. indukt. Abstamm.-u. Vererbungslehre **45**, 279 (1927). The author wishes to express his thanks to Dr. Judd for making available a photostatic copy of this paper.

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 33, NUMBER 12 DECEMBER, 1943

Proceedings of the Twenty-Eighth Annual Meeting of the Optical Society of America

MELLON INSTITUTE AND HOTEL WEBSTER HALL, PITTSBURGH, PENNSYLVANIA, OCTOBER 7–9, 1943.

THE outstanding features of this meeting were the large number of contributed papers on the problems of applied spectroscopy and the unusually large attendance. In spite of wartime limitations on travel and on the subject matter of papers, as well as the absorption of scientists in the war effort, a total of 395 persons registered. The sessions were held in the Auditorium and in the Fellows Room of the Mellon Institute, while nearby Hotel Webster Hall served as headquarters for the meeting.

CONTRIBUTED PAPERS

Sessions for the presentation of contributed papers began on the afternoon of Thursday, October seventh, at which time a group of papers on color was presented in the Fellows Room. Simultaneously, a series of papers on spectroscopy was presented to a large group in the Auditorium. Again, on the afternoon of Friday, October eighth, there were simultaneous sessions: the one being a group of papers on general optics; and the other, a group of papers on spectroscopy. On Saturday, October ninth, there was a morning session and an afternoon session for the presentation of papers on spectroscopy. In all, thirty-seven papers were presented at the meeting, the authors' abstracts of the contributed papers being appended.

TRIP TO ALLEGHENY OBSERVATORY

On the evening of Thursday, October seventh, three large chartered trolley cars took capacity loads from the Hotel Webster Hall to the Allegheny Observatory, where members of the staff had arranged a program enabling the visitors to see some of the experimental research and the routine work of the observatory. Opportunity was given to view the moon and other objects through the large telescopes. After the demonstrations, the guests assembled for refreshments and a social hour.

INVITED PAPERS ON ASTRONOMICAL OPTICS

On Friday morning, October eighth, a group of invited papers was given in the Auditorium of the Mellon Institute.

Dr. H. D. Babcock's paper, entitled "Bright diffraction gratings," was presented by Dr. Nicholas Wagman in the unavoidable absence of the author. The history of gratings was reviewed briefly, leading up to recent development of gratings that give a high intensity in a specified order on one side. The theory of geometrical reflection from the plane sides of unsymmetrical grooves permits calculation of the cross section of groove when grating space, order of spectrum, wave-length, and angle between collimator and camera are given. The ruling diamonds are cut and polished on a special machine that produces highly polished convex surfaces intersecting at a specified angle. Various metals and some alloys have been studied in castings, rolled plates, evaporated and electrolytic films. Height of maximum in the diffracted beam is greater and its angular width much less as the spacing of the grooves becomes greater. The percentage of the incident monochromatic light of wave-length $\lambda 5461$ found in the emergent beam is 60 percent for 600 grooves per millimeter, 70 percent for 300 lines per millimeter. These are nearly the same as for combinations of dense flint 60° prisms giving comparable dispersion. A transmission replica with 30 grooves per millimeter gives 85 percent.

Dr. R. Minkowski's paper on "Schmidt systems as spectrograph cameras" was read by Dr. Rudolf Kingslake. Schmidt systems are particularly suited for use as spectrograph cameras. For focal lengths above 30 inches, photographic plates can be bent to fit the focal surface. For shorter focal lengths, curvatures necessitating the use of film can be avoided by the introduction of a field-flattening lens. A plano-convex lens placed close to the focus has been completely satisfactory in cameras of focal ratio $f/3$ and $f/1.5$. Flat plates without field flattener suffice for many purposes.

Loss of light due to occultation by the plate can be avoided by an off-axis construction. But the usefulness of field-flattening lenses in off-axis systems will be restricted to focal ratios of $f/3$ and larger. The use of flat plates in off-axis systems results in a sidewise curvature of the spectrum on the plate. Such a curvature is objectionable, and off-axis systems of short focal length are of restricted usefulness.

For cameras of very short focal length, the small scale of the spectrum permits the use of small plates, thus minimizing the loss of light in a normal system. But a range of focal lengths below 30 inches will always remain for which the full advantages of Schmidt systems are not available without loss of light.

In his paper, "The McGregor spectrograph of the McMath-Hulbert Observatory," Dr. Orren C. Mohler described this recently completed addition to the equipment for solar research. One of the ultimate goals set by this observatory is the complete observational description of a solar prominence. This will require simultaneous exposures with a number of instruments. The new instrument is one of a group used in this way. It was designed by Dr. Robert R. McMath, Director of the Observatory, to provide the necessary measures of the intensity of the light coming from any and all parts of a solar prominence.

The new instrument is of the vertically mounted Littrow type used with tower telescopes. The first and second slits, together with their actuating mechanisms, and the plate holder are mounted conveniently in a rotatable spectrograph head that forms a table on which the image of the sun is focused. The light, passing through the first, or entrance, slit, proceeds downward to the lens-and-grating unit from which it is returned through the second slit to photoelectric devices or to the plate holder. The instrument possesses several unusual features demanded by the nature of the observations. The grating has 600 lines per millimeter and is ruled for maximum intensity for H alpha in one of the first-order spectra. The focal length of the lens is 25 feet. Grating and lens are mounted as a unit capable of rotating 360° about an axis joining the center of the first slit and the center of the grating, and are mechanically independent of the spectrograph head. The entrance slit and the second slit, used to limit the wave-length band emerging from the instrument, are mounted in two sets of perpendicular ways on the spectrograph head. They are movable by cams so that each point on the slits sweeps through an area 1×1 inch. Thus the first slit scans whatever is imaged upon it; the light enters the spectrograph, is dispersed, and emerges from the second slit to impinge on a photo-cell or other light measuring device. By automatically recording the photoelectric current on a conventional photographic tape recorder, it is possible to find the light intensity at any point within the scanned area. Other important features are: spectrum centering by electric motor; spectrum drift (wave-length change) by selsyn motor with direct reading wave-length dial; location of all controls at the spectrograph head; and provision for rapid interchange between photoelectric and photographic use of the instrument. By way of illustration of some of the problems encountered in this work, about 20 minutes of recent motion picture films of solar phenomena were shown to an appreciative audience.

The Annual Dinner

Three hundred and twenty-five persons attended the annual dinner of the Society on the evening of Friday, October eighth, in the main dining room of the Hotel Webster Hall. President Worthing expressed the thanks of the Society to all those whose efforts had made the meeting so successful. Then, after presenting the newly elected officers, he called for a few non-spectroscopic remarks from the spectroscopists. These post-prandial efforts were terminated in ample time for all to reach their seats for the evening lecture.

Award of the Frederic Ives Medal

After the Society dinner on the evening of Friday, October eighth, the Society convened in the Auditorium of the Mellon Institute. Dr. Worthing, presiding, called upon Dr. W. E. Forsythe, Chairman of the Committee of Award for the Frederic Ives Medal. Dr. Forsythe traced briefly the history of the medal, how it is awarded for distinguished work in optics, named the earlier recipients, and then presented Dr. Loyd

A. Jones as the Committee's choice for the award. After the formal presentation of the medal by President Worthing, the Medalist responded briefly, expressing his appreciation of the honor, and then delivered the lecture of the evening on "Psychophysics and photography."

Business Session

There was a brief business meeting, with President Worthing in the chair, on Friday morning, October eighth, just before the reading of the invited papers on Optics in Astronomy. The annual reports of the Secretary, the Treasurer, and the Editor of Publications were accepted and filed. Dr. H. A. Barton, Director of the American Institute of Physics, then reported on the progress being made in acquiring a permanent home for the Institute, stating that contributions had been received for two-thirds of the amount needed. He called the attention of all to the need for additional contributions and pointed out the advantages accruing from an early completion of the financing. Dr. Barton also spoke of some recent changes in the machinery for the deferment of scientific and technical men in the draft, indicating that these changes are in a direction likely to hinder rather than to help physicists as a group in rendering their special services to the nation in connection with the war.

Meeting of the Board of Directors

The Board of Directors held a meeting on Thursday, October 7, 1943. The minutes of this meeting are appended.

Contributed Papers on Color

1. The Subjective Boundaries of the Spectral Colors. Lieutenant Dean Farnsworth, *Medical Research Laboratory, Submarine Base, U. S. Navy.*

Arbitrary divisions of the spectrum have been made for convenience in distinguishing color regions of the spectrum for centuries but without controlled methodology and without notable agreement. A few boundaries have been established for special conditions of observation; other studies have been limited by physical or perceptual factors in the presentation of the spectrum. This experiment used 400 color-naive observers of various ages, occupations, and educational backgrounds and a small number of color-experienced observers; a projected spectrum was presented to the light adapted eye; the surround was large, neutral and of moderate brightness; effects of induced contrast were controlled by automatically timed rests. Instructions were to "set the needle between the colors—so as to divide one color from the next;" data consisted of readings on a calibrated scale of the positions at which the observers set the needle; instructions were given and settings were made without the use of color names. Boundaries are presented for three-, four-, five-, and six-color divisions. There was excellent agreement on these boundaries among the judgments of the 400 naive subjects but not among the judgment of the color specialists, artists, psychologists, and other theory-indoctrinated observers. The means of the settings of the naive observers are presented as divisions which would specify valid and generally acceptable spectral regions for use in commerce, science and education.

2. Quantitative Formulations of Color Mixture and Wave-Length Discrimination Data for Dichromats. Glenn A. Fry, *Ohio State University, School of Optometry.*

When matches for the spectral colors are determined by dichromats by mixing two primaries at the red and blue ends of the spectrum, the energy ratios required for the matches can be expressed in terms of the following equation:

$$\log R_E = K(\lambda - C).$$

In this equation R_E represents the energy ratio of the two primaries, lambda represents the wave-length matched, and K and C represent constants. The value of K remains relatively constant from one dichromat to another. The C value for the protanope is lower than that for the deuteranope, and the difference cannot be attributed to the protanope's reduced brightness response for wave-lengths at the red end of the spectrum; it has to be accounted for in terms of valence relationships between the primaries. The relative valence of two stimuli has been arbitrarily defined by assuming that the two stimuli have equal valence when there are just as many J.N.D. steps from a half-and-half mixture to one pure component as to the other pure component. Wave-length discrimination data have been used to determine the energy ratio of the two primaries required to produce equal valence. When the energy ratio for the two primaries is adjusted to give them equal valence it is found that an approximately linear relationship exists between the J. N. D. steps and the percentage of yellow or blue in the di-stimulus matches for the various wave-lengths. The above formula for color mixture data for dichromats can be used in determining the di-stimulus matches for various heterochromatic stimuli. This procedure can be applied in connection with the determination of confusion colors for dichromats.

Bibliography:
1. S. Hecht and S. Shlaer, "The Color Vision of Dichromats," J. Gen. Physiol. **20**, 57–93 (1937).
2. F. H. G. Pitt, "Characteristics of Dichromatic Vision," Medical Research Council, Special Report, No. 200 (1935).

3. Chromatic Valence as a Correlate of Munsell Chroma. ELLIOT Q. ADAMS, *General Electric Company, Nela Park, Cleveland.*

By applying the Munsell-Sloan-Godlove value function to all three tristimulus values, X_C, Y, and Z_C, and plotting the differences, $V_X - V_Y$, $V_Z - V_Y$; with a scale-factor of $2\frac{1}{2}$, radial distances from the illuminant point were found to be reasonably well correlated with Munsell chroma along any one hue radius, but equal differences in chroma corresponded to quite different distances, e.g., for the hues Y and PB, on the "chromatic value" diagram. This calculation assumed that the sensitivity of each of the three sets of retinal receptors is independently determined by the respective tristimulus value. If it be assumed that the sensitivity of all three sets depends on the same mean between the general and local values of Y, the functions, $[(X_C/Y)-1]V_J$, $[(Z_C/Y)-1]V_J$, result. V_J signifies that the Judd (quintic-parabola) value function has been used in the calculations. These functions, collectively called "chromatic valence," plotted with a scale-factor of $2\frac{1}{2}$, give nearly equal radial distances for equal changes in Munsell chroma, e.g., for the hues Y and PB. Since the functions have the common factor V_J, their ratio, $(X_C - Y)/(Z_C - Y)$ is the same for all colors of the same dominant wave-length. For colors of equal value the "chromatic valence" diagram is geometrically similar to the constant-brightness chromaticity diagram.

BIBLIOGRAPHY:
E. Q. Adams, J. Opt. Soc. Am. **32**, 168–173 (1942).
S. M. Newhall, D. Nickerson, and D. B. Judd, J. Opt. Soc. Am. **33**, 385–418 (1943).

4. A New Formulation of Color Harmony. PARRY MOON, *Massachusetts Institute of Technology.*

Numerous theories of color harmony have been formulated but the results have not been satisfactory, partly because of difficulties with the specification of color and partly because of the neglect of the relative areas of the color patches. Before the advent of the C.I.E. system, scientific color harmony was barely possible; and even the C.I.E. color space cannot be used directly in color harmony because equal perceptual steps are represented by quite unequal distances. This paper develops a system of color harmony based on the metric color space considered in a previous paper.[1] The theory makes use of ideas of harmony found by the artist but states them quantitatively. The system is based on two ideas: (1) colors should be spaced so that ambiguities are eliminated, (2) the points representing these colors in ω-space[1] should be related in a simple geometric manner. On this foundation, color harmonies are classified, starting with the simplest two-color achromatic harmonies and progressing to complicated harmonies in color 3-space. The adjustment of area to obtain an aesthetic effect is also considered. A number of examples are given to show the procedure in using the new system of color harmony.

[1] J. Opt. Soc. Am. **33**, 270 (1943).

5. Generalized Theory of Color Harmony. DOMINA EBERLE SPENCER, *Tufts College, Medford, Massachusetts.*

In the paper "A New Formulation of Color Harmony"[1] a fixed state of adaptation is assumed. That is, the eye is assumed to be adapted in all cases to a neutral gray of Munsell value 5. But this is by no means the most general case. Adaptation may be to a chromatic surround or may be influenced by the design itself. The general case will be treated in the present paper. The basis for the theory is the metric colorspace[2] previously developed and the composite stimulus.[3] Postulates are formulated which define a harmony with any state of adaptation. The mathematical details of obtaining satisfactory harmonies for any given adaptation are then worked out. The more difficult cases where adaptation is determined by the design itself are then treated. An interesting application of the method is to the theoretical investigation of the effect of the illuminant on the aesthetic value of a colored design.

[1] Preceding abstract.
[2] J. Opt. Soc. Am. **33**, 270 (1943).
[3] J. Opt. Soc. Am. **33**, 10 (1943).

6. Aesthetic Measure in Color Harmony. DOMINA EBERLE SPENCER, *Tufts College, Medford, Massachusetts.*

In all previous formulations of color harmony, there has been no attempt to specify which harmonies are good and which are poor. Certainly some of the recommended color combinations are much better than others. The purpose of this paper is to consider a rating system for harmonies. The general subject of aesthetic measure gives a numerical designation to the aesthetic qualities of any form of art. G. D. Birkhoff developed the subject, and in his book, *Aesthetic Measure*,[1] applied it to polygonal shapes, to music, and to poetry. He did not apply the subject to color harmony. This paper extends Birkhoff's work to color harmony. The aesthetic measure is defined as the ratio of the elements of order to the elements of complexity. The constants used in evaluating these elements were determined by experiments on colored designs. By use of these constants it is now possible to rate any design on the basis of elements of identity, similarity, contrast, ambiguity, complexity, and area balance. Examples are given showing how change in color alters the aesthetic measure of a design.

[1] G. D. Birkhoff, *Aesthetic Measure* (Harvard University Press, Cambridge).

Invited Papers on Astronomical Optics

7. Bright Diffraction Gratings. HAROLD D. BABCOCK, *Mount Wilson Observatory.*

8. Schmidt Systems as Spectrograph Cameras. R. MINKOWSKI, *Mount Wilson Observatory.*

9. The McGregor Spectrograph of the McMath-Hulbert Observatory. ORREN C. MOHLER, *The McMath-Hulbert Observatory.*

Contributed Papers on Optics

10. Gaussian Optics and Gaussian Brackets. MAX HERZBERGER, *Kodak Research Laboratories.*

Gauss invented an algorithm for certain arithmetical purposes, which has various applications in optical problems. For instance, it is sometimes of value to see how focal length, magnification, and back focus of a lens system vary with the change of one of the construction elements (thickness, radius, or refractive index). It is sometimes very useful to have explicit expressions for these significant lens characteristics as functions of the construction element. However, the usefulness of these Gaussian brackets is not limited to Gaussian optics. The image errors for finite rays in the new form recommended by the author can be expressed with their help.

11. A New Optical Bench. NORMAN F. BARNES, *General Engineering Laboratory, General Electric Company.*

For precision optical set-ups using standard laboratory equipment, the straight-line type of optical bench has two disadvantages. First, its use is very impractical for systems whose axes do not form a single straight line; and second, precision work on this type of bench even with straight-line systems is difficult, since the usual adjustment available for a given bench clamp position is only a vertical one. Hence, in order to get three-dimensional adjustments, it is often necessary to use a maze of clamps mounted on the bench riders as well as additional clamps to hold external stands in their proper positions. A new optical bench which obviates these difficulties consists of a steel base plate upon which are mounted powerful Alnico magnets into which are inserted vertical rods. These stands can be rotated about a vertical axis or easily slid to any desired position upon the base plate, at the same time holding themselves firmly to the plate. An additional advantage of the system is that it can be used wherever a steel base is obtainable. Thus, for example, a system can be set up on a vertical steel panel wall or upon a steel plate held in a vertical position.

12. An Optical Sighting Gauge for the Garand Rifles. C. B. SITTERSON, JR., AND NORMAN F. BARNES, *General Electric Company.*

Although the trial-and-error method of firing rifles in order to adjust their sights is unimpeachably accurate, the method is slow, and the necessary material and equipment are very expensive. Furthermore, the shortage of labor accentuates the need for a more rapid method, and the exigencies of war demand a more effective use of the ammunition. Investigation showed that the last few inches of the bore of the rifle substantially determines the projectile trajectory. Thus the rifle sights can be adjusted by aligning them with respect to these last few inches of the bore. The process involves two main steps. First, the last few inches of the bore must be accurately aligned in space; and second, the sights must be adjusted to their correct relationships with respect to this alignment. Into the muzzle of the rifle is inserted a steel plug supporting a concave mirror, the latter imaging a fixed cross of light upon an indexed screen. The position of the cross image is determined by the orientation of the concave mirror and is so adjusted that the cross image coincides with a marked screen position. This is done by pivoting the rifle about a point in space located at the muzzle end of the bore center. Two projectors showing a 25X magnified image of each of the two sights indicate the locations of these sights relative to the positions required for correct alignment. It is then a simple matter to adjust the sights mechanically to their correct positions and set the index.

13. Displacement of the Focal Point of a Lens System Resulting from Change of Temperature. ROBERT E. STEPHENS, *National Bureau of Standards.*

An expression for the displacement of the focal point resulting from changes in indices of refraction and radius of curvature for a single spherical refracting surface is found by differentiation of the equation

$$n'/s' = n/s + (n' - n)/r.$$

By summing such expressions and considering changes in thicknesses and spacings, an expression is found for the displacement of the focal point of a complete centered lens system as a function of the changes in indices of refraction, radii of curvature and spacings. This expression may be used for computing the displacement resulting when the indices, radii and separations are changed by a variation in temperature. It is not restricted to this use however, since it is valid for small changes in the independent variables from any cause whatever.

14. The Design of a Telephoto-Magnifier. MAX REISS, *Eastman Kodak Company, Rochester.*

As far back as 1851 Brücke[1] pointed out the need for a magnifier of rather high magnification but large object distance. In fact there are many cases where such a lens is useful: e.g., for observation of anatomical specimens, as a loupe for eye physicians, for the inspection of mechanical parts, etc. In all these cases the use of tools or an illuminating lamp requires a considerable space between the last lens surface and the object under observation. By combining a lens of low magnification and therefore large object distance with a telescopic system, the magnification is increased by a factor equal to the angular magnification of the telescopic system, while the large object distance remains constant. Replacing the positive eyepiece of the telescopic system by a negative one reduces the over-all length of the system considerably and also secures an erect image. An additional advantage of using a negative eyepiece is the possibility of satisfying the Petzval-condition and thus covering a wider field without astigmatism. The complete system is thus a reversed telephoto lens characterized by a small focal length and large object distance. Starting from a doublet and solving for the required power, back focal length, and Petzval sum, the system arrived at showed heavy aberrations and turned

out to be too simple for the stated requirements. The considerable distance between the elements of the lens leads to high angular deviations and therefore large aberrations. Furthermore, in order to secure a fairly large field with comfortable vision comparatively wide pencils and therefore large lenses have to be used. This also causes large aberrations. The proper analysis of the system on the basis of the Seidel contributions, and also by using the data of trigonometrical ray tracing for determining the contributions of the individual surfaces to the final aberrations, leads to an expansion of the system and points the way for systematically improving the lens step by step until all the aberrations are reduced to tolerable amounts. Because of the necessity of using wide pencils special care had to be given to the correction of the oblique pencils. The focal length of the final lens was 41 mm, the back focal length 62 mm, and the angular semi-field about 12°.

[1] E. Brücke, Wiener Sitz. 6. 554 (1851).

15. The Geometric Identification of Reflection and Transmission Measurements. RICHARD S. HUNTER, *National Bureau of Standards.*

To identify any method for measuring reflection or transmission, its spectral and geometric characteristics must be given. By geometric characteristics are meant the directions in which light is allowed to strike specimens and the directions in which light reflected or transmitted by them is accepted for measurement. There are three types of reflection measurements which must be identified geometrically:

TERM	SYMBOL	DEFINITION
Reflectance (Total)	R	Total light reflected by specimen divided by that incident on it.
Fractional reflectance	R_f	Light reflected by specimen in specified directions divided by that incident on it.
Directional[1] reflectance	R_d	Light reflected by specimen in specified directions divided by that which would be reflected in same directions by perfectly diffusing, completely reflecting specimen.

The suggested term, fractional reflectance,[2] is new. The suggested terms, symbols and definitions for the three corresponding types of transmission measurements are identical except for the substitution of the words transmittance[3] and transmitted, for reflectance and reflected, respectively, and for the use of the letter T instead of R. The directions of incidence must be included in descriptions of all three types of quantities and the directions of viewing in descriptions of the latter two. In some descriptions, it is necessary to give only the central directions, but where the powers of specimens to reflect or transmit light may change importantly within the directions involved, the solid-angle segments of directions must also be given. Unless an instrument is free of vignetting, moreover, the weights given to incident and reflected light within the individual components of these segments must become a part of the complete geometric description. It is shown that devices to measure specular gloss should be identified by complete geometric descriptions and that some such devices heretofore considered geometrically identical may differ because of differences in spread or vignetting. It is also shown that if the receptor is small and uniformly sensitive, values for identical geometric conditions on the second and third scales above are related by the equation

$$R_f = \frac{\omega \cos \theta}{\pi} R_d$$

where ω is the solid angle of the receptor segment and θ is the angle between the normal to the test specimen and the axis of this segment.

[1] Called apparent reflectance by H. J. McNicholas, "Absolute Methods in Reflectometry," Bur. Stand. J. Research 1, 29 (1928) RP3. Present term proposed by O. S. A. Colorimetry Committee, report now in preparation.

[2] Called specific reflectivity by H. C. Hamaker, *Reflectivity and Emissivity of Tungsten* (N. V. Noord-Hollandsche Uitgeversmaatschappi, Amsterdam, 1934).

[3] Heretofore called transmission, but termed transmittance in the present draft of the Report of the O. S. A. Colorimetry Committee.

Contributed Papers on Spectroscopy

16. A Compendium of Operating Conditions Used in Spectrographic Analysis. EDWIN S. HODGE, *Harry W. Dietert Company.*

The recent growth in interest and use of emission spectroscopy has increased the demand for information regarding quantitative analysis. Much valuable information has been published, but frequently it is inaccessible to the busy industrial worker. To make this material more generally available the literature from 1920 to 1942 has been carefully surveyed and this paper gives a preliminary report of the work thus far. The Bibliography of Meggers and Scribner[1] has been used as a key and all their citations as well as more recent ones have been examined by title, by abstract, or in the original source. The data contained in the original have been carefully examined and when the internal standard procedure, or the homologous pair technique has been used, the line pairs and listed operating conditions have been recorded in appropriate tables. Abstracts from several important articles on magnesium analysis will be distributed as examples of how the work is being collected. Over 500 articles in the original journals are being examined and essential information recorded. It is expected that the information will be made available by publication. As a result of this survey the incompleteness of many reported methods is obvious, some suggestions are given on the importance of listing all the desirable information when reporting a quantitative method. Any operating condition and line pairs being used in industrial laboratories will be gratefully received and acknowledged.

[1] W. F. Meggers and B. F. Scribner, *Index to the Literature on Spectrochemical Analysis. 1920–1939*, second edition (American Society for Testing Materials).

17. Sampling, Mixing, and Grinding Techniques in the Preparation of Samples for Quantitative Analysis by X-Ray Diffraction and Spectrographic Methods. JAMES W. BALLARD, H. I. OSHRY, AND H. H. SCHRENK.

Methods of sampling for quantitative analysis by x-ray diffraction and spectrographic methods are described. For gross material a "diamond" mortar and pestle, ore grinder, mixing clothes and riffle are used for obtaining a small representative sample. For x-ray diffraction analysis a small portion of this material is mixed with an internal standard and reduced to homogeneous mixtures of satisfactory particle size range in ball mills or in a motor-driven mortar and pestle. A ball-mill technique for mixing samples for spectrographic analysis is described which minimizes the effects of sample inhomogeneity on quantitative analyses. The methods, although applied to specific problems, may be of use in other techniques where a very few milligrams of material must be made to represent a large gross sample.

18. Spectrographic Determination of Thorium in Tungsten Filament Wire. S. L. PARSONS, *Sylvania Electric Products Inc.*

It is rather well known that a monatomic layer of thorium on the surface of a tungsten filament greatly enhances the electron emission. In the manufacture of vacuum tubes for transmitting and other high power applications it is necessary to control the thorium content of the filament wire so that uniform processing schedules can be set up. As the thorium is added to the tungsten powder before it is fused and drawn into wire, there are many opportunities for losses which affect the final concentration. An attempt was made to determine the thorium content spectrographically, as the chemical analysis is rather slow due to the difficulty of putting tungsten into solution. Several size filaments are used in our plant, varying from 0.004" diameter to 0.013" diameter. It was found that reproducible spectra could be obtained using a 2200 volt a.c. arc between electrodes prepared by inserting short lengths of wire into a small hole drilled in each graphite electrode. Using a series of chemically analyzed standards, an analytical curve was made up for thorium over the range 0.9 percent to 2 percent. The precision of the method was adequate for our needs; the standard deviation from the mean of a set of 84 determinations on a sample containing 1.5 percent thorium, was about 3.5 percent. This analysis has been in use now for about a year, every coil of wire being checked before going into production. About 75 samples per month are submitted so a high speed analysis setup has not been necessary.

19. Spectrographic Analysis of Magnesium Alloys. T. M. HESS AND L. G. REINHARDT, *The Dow Chemical Company, Midland, Michigan.*

The spectrographic method of analysis of magnesium alloys in use in this laboratory is described. The condensed spark discharge is used for the analysis of magnesium alloys for the alloying constitutents and certain residual impurities. The arc discharge is used in analysing for such impurities as iron and nickel. The influence of such factors as microporosity and grain size on the analytical results is considered. The effects of electrode size and shape, discharge area, gap length and pre-spark time are discussed. Problems encountered in the sampling of large castings are also considered.

20. Spectrographic Determination of Trace Elements in Cartridge Brass and Admiralty Brass. P. A. LEICHTLE, *Chase Brass & Copper Co., Inc.*

A spectrographic procedure for the determination of Pb, Fe, Ni, Sn, Bi, and Sb in Cartridge Brass and Admiralty Brass is described in detail. The source is a 14 amp. low voltage d.c. arc of conventional type. A 50-mg pellet sample is arced to extinction in a cupped graphite electrode of specified dimensions. The intermediate mode of illumination obtained by focusing the source on the collimator lens is adopted. The light intensity is reduced by means of a rotating radial sector placed at the slit. The step sector method of plate calibration is used. The calibration source is a 4 amp. d.c. arc on $\frac{3}{16}$" diameter copper electrodes. Working curves are given for the determination of Pb, Fe, Ni, Sn, and Sb in the range of 0.01–0.10 percent and Bi in the range of 0.0004–0.010 percent. The method requires the use of standard samples for preparation of the working curves.

21. The Spectrographic Determination of Small Amounts of Impurities in Magnesium Chloride. MABEL F. WILSON, *Diamond Alkali Company.*

Rigid control of small amounts of impurities in magnesium chloride is necessary for the successful electrolytic deposition of magnesium metal. Chemical determinations of these impurities are often difficult, time consuming, and of questionable accuracy. To increase both speed and accuracy, a quantitative spectrographic method has been worked out and is being used in routine analysis. Purified graphite electrodes are coated with the diluted liquor to which has been added a definite amount of a suitable internal indicator. The dried electrodes are subjected to a.c. arc excitation, two exposures being sufficient to cover the full range of impurities. Carefully controlled conditions of development make the calibration of each plate unnecessary. Background corrections are made where a longer exposure is required. Analytical curves for boron, silicon, iron, manganese, aluminum, copper, sodium chloride, and calcium chloride are in daily use. Discussion of spectrographic equipment, development and densitometric procedure, methods of calculation and typical results obtained will be presented in this paper.

BIBLIOGRAPHY:
1. O. S. Duffendack, and K. B. Thomson, Proc. A. S. T. M. **36**, 301 (1936).
2. Cholak, Jacob and Story, J. Opt. Soc. Am. **31**, 732 (1941).
3. Malpica, Steele, and Mulvey, A. S. T. M. Symposium, 1940.
4. W. C. Pierce, and N. H. Nachtrieb, Ind. Eng. Chem. Anal. Ed. **13**, 774–781 (1941).
5. J. T. Mireles Malpica, Gen. Elec. Rev. **43**, 288–97 and 333–5 (1940).
6. F. Twyman, J. Inst. Metals **64**, Advance copy No. 828 (1939).
7. W. Gerlach and E. Schweitzer, *Foundations and Methods of Chemical Analysis by the Emission Spectrum* (London, 1931), p. 67.
8. W. R. Brode and E. S. Hodge, J. Opt. Soc. Am. **31**, 58–63 (1941).
9. Jacob Cholak and R. V. Story, J. Opt. Soc. Am. **32**, 502–5 (1942).
10. A. C. Coates and E. H. Amstein, J. Soc. Chem. Ind. **61**, 24 (1942).
11. Cholak, Hubbard, and Story, Ind. Eng. Chem. Anal. Ed. **15**, 57–60 (1943).
12. A. G. Quarrel and G. E. A. Bramley, J. Inst. Metals, **67**, paper No. 891, 25–41 (1941).

22. Direct Reading Instrument for Spectrochemical Analysis. M. F. HASLER, *Applied Research Laboratories, Glendale, California,* AND H. W. DIETERT, *Harry W. Dietert Company, Detroit, Michigan.*

The conventional method of quantitative spectrochemical analysis employing bright line spectra and photographic recording is discussed with reference to the various steps involved. The steps which may be eliminated by the application of direct photoelectric recording are considered and a method based on this type of recording is discussed. A detailed comparison of the two methods is made which leads to the conclusion that only by designing an entirely new spectrum dispersing instrument can "direct reading" be made sufficiently flexible for general adaptation. Such an instrument is considered in detail together with suitable analytical procedure for its use. The probable speed and accuracy attainable are illustrated by examples. Present limitations and future developments are considered. A proposed instrument is announced.

23. The Application of Multiplier Photo-Tubes to Quantitative Spectrochemical Analysis. E. A. BOETTNER, *Wyandotte Chemicals Corporation, Wyandotte, Michigan* AND G. P. BREWINGTON, *Lawrence Institute of Technology, Detroit, Michigan.*

A bridge circuit consisting of two multiplier photo-tubes (RCA 931) has been adapted to indicate the ratio of intensities of spectral lines when the tubes are mounted on a spectrograph in place of the photographic film. It has been found that holding the voltage constant on one tube (set on an internal standard line) and varying the voltage applied to the other tube (set on an impurity line), the plot of the logarithm of the concentration of the impurity in the source *vs.* the difference of voltage applied to the two tubes is a straight line. Quantitative analyses made using this circuit and calibration method have approached and in some cases equaled the accuracy and sensitivity possible with a photographic plate. Some indication has been obtained that if proper light sources were available the galvanometer reading representing unbalance of the bridge might be of significance for quantitative analysis.

24. The Measurement of Spectral Intensities in the Direct-Current Arc. ROBERT R. HAMPTON WITH H. N. CAMPBELL, *United States Rubber Company.*

A study has been made of the effects of various factors (other than sample composition) on the reproducibility of spectral line intensities in the low voltage direct-current arc between graphite electrodes. A definite weight of cobalt oxide was arced to provide an external standard for the step sector calibration of each plate. Measurement was made of the variations in intensities of typical arc and spark lines due to variation in: (1) portion of the arc from which light is taken, (2) arc gap length, (3) position of the arc (wandering), and (4) arc current. It is concluded that, when light is taken only from the positive column of the arc, and the current is held constant by an automatic regulator, precision is much better than usually reported for arc methods. Photographic photometry is discussed with special reference to the advantages of using low contrast plates. Some typical analytical working curves are presented and the advantages of the external standard method are shown, in comparison with the internal standard method. A simple procedure has been developed for the determination of traces of metals in organic materials. A weighed sample (pelleted, if solid) is ashed directly in an electrode containing a lithium-strontium buffer mixture, and then arced. Intensities are measured as above.

25. Accessory Equipment for Spectrochemical Analysis. J. L. SAUNDERSON AND V. J. CALDECOURT, *The Dow Chemical Company, Midland, Michigan.*

This paper describes miscellaneous accessory equipment used in this laboratory for the spectrochemical analysis of magnesium alloys. Consideration of the purpose of the equipment here described shows that rigid control of the experimental conditions during the analysis as well as convenience and efficiency of operation dictate the design of equipment. The equipment to be described will include electrode clamps for routine use, an automatically operating arc starter, electrode cutting and shaping apparatus, a system for automatic timing of exposures and operation of a spectrograph, a stabilized power supply for a microphotometer requiring a large current at low voltage, and an intensity control sector.

26. A Preliminary Curve for a Spectrographic Plate Calibration Curve. H. S. READ, *Eastern Representative, Harry Dietert Company.*

A plate calibration shows the relation between known steps of relative intensity of light source and meter readings of the microphotometer. A log step sector is often used to provide known steps of intensity along the slit and resultant steps of exposure along the photographed slit image. To cover a 1–100 range from transparency to blackness of lines useful in spectrography eight 1 : 2 steps or thirteen 1 : 1.5 steps are needed. Commercially available step sectors do not provide so many steps. If they did, uniform illumination and photography would be very difficult or impossible. To reduce these difficulties, the author has developed a preliminary curve method. With it only two steps, i.e., one pair of exposures is needed. Sufficient uniformity of lines is easily attained. The preliminary curve is used to develop a table, i.e., a synthetic step sector pattern for the whole range of relative intensities. With this the well known plate calibration curve can be plotted accurately. Original readings, procedure, and curves will be shown to demonstrate this useful method of making a plate calibration curve.

27. An Improved Spectrochemical Method Using Spark Excitation. HENRY A. SLOVITER and ALEXANDER SITKIN, *Test Laboratory, U. S. Navy Yard, Philadelphia.*

A method of quantitative spectrochemical analysis is described in which the sample to be analyzed is dissolved in a suitable medium and brought to a desired concentra-

tion. Two flat-top, treated graphite electrodes are then impregnated with this solution and allowed to dry. These electrodes are then excited in a simple, condensed spark circuit and the spectra are recorded by a spectrograph without using a condensing lens. The usual scheme of internal standard analysis is then applied, using a calibrated photographic plate. Data are given which indicate good reproducibility of excitation conditions when various steels are analyzed by this method. Working curves for the determination of manganese up to 1.5 percent, for nickel up to 0.5 percent, and for chromium, vanadium, molybdenum, and copper up to 0.3 percent are shown. The advantages of such a scheme of analysis over the commonly used one involving the use of solid pieces of the sample as the electrodes are:

(1) The effect of the metallurgical condition of the sample on the behavior of the electrodes in the excitation circuit is eliminated.

(2) The absolute concentration of material in the electrode can be adjusted at will and foreign elements with favorably situated lines can be readily introduced.

(3) The source of error due to local segregation in a solid sample used as electrode is eliminated.

(4) Standard solutions to be used for construction of working curves can be easily prepared from pure materials.

The results of studies of the behavior of such an electrode system during excitation are also given.

BIBLIOGRAPHY: Scheibe and Rivas, Angew. Chem. **49**, 443 (1936).

28. Spectrochemical Analysis of Metals with the Multi-Source Unit. M. F. HASLER AND J. W. KEMP, *Applied Research Laboratories, Glendale, California.*

The various factors that must be considered in a spectral source suitable for quantitative spectrochemical analysis by means of bright line spectra are considered. These include precision, sensitivity, range, and the effect of one element upon another's quantitative determination. Thermal equilibrium arc sources, non-equilibrium spark sources and intermediate cases are discussed with respect to these factors. The wide variety of discharge conditions possible with the multisource unit is turned to practical application. Steel, aluminum, brass, magnesium, and zinc are each analyzed using several different types of discharges. The results obtained are given in detail and discussed on the basis of the factors mentioned above. From both theory and practice it is apparent that no single best discharge condition exists for the analysis of a variety of elements in different types of alloys. Each practical problem demands a compromise solution particularly if a single exposure must be used to obtain the complete analysis. The multisource with its great range of discharges allows the most suitable conditions to be utilized in each case.

BIBLIOGRAPHY: M. F. Hasler and H. W. Dietert, J. Opt. Soc. Am. **33**, 218–228 (1943).

29. Accuracy in the Spectrographic Analysis of Steel. BOURDON F. SCRIBNER AND CHARLES H. CORLISS, *National Bureau of Standards.*

Several papers[1,2] have discussed the probable accuracy of the spectrographic analysis of steel and recently an extensive statistical treatment of precision in the determination of chromium has been presented.[3] In an investigation of steels issued as spectrographic standards by the National Bureau of Standards, various factors bearing on the ultimate accuracy of analyses with the standards were studied. The factors of primary importance in the reliability of standards are homogeneity and accuracy of the chemical analyses. The homogeneity of the steels varied markedly with the constituent, the order of decreasing reproducibility being: Cr, Mo, V, Mn, Ni, Si, and Cu. The accuracy of chemical analyses was estimated, in general, to be inferior to the spectrographic precision below concentrations of 0.5 percent and superior above 1 percent. In addition to the integrity of the standards, the accuracy of determinations based on the standards depends on the manner in which the standards are employed. Several variables in the spectrographic procedure, including excitation conditions, choice of line pairs, photometric measurement, and effects of variation of matrix of the steel were investigated. The results indicate that uncertainties may be introduced in analyses that will not be evident in the measurements of reproducibility in isolated cases. Precautions to be observed in the application of standards will be discussed in the light of these observations.

[1] F. G. Barker, J. Iron and Steel Inst. **139**, 211 (1939).
[2] H. B. Vincent and R. A. Sawyer, Metal Progress **36**, 35 (1939).
[3] H. B. Vincent and R. A. Sawyer, J. Opt. Soc. Am. **32**, 686 (1942).

30. The Use of Briquets Formed from Metal Grindings for the Spectrographic Analysis of Steel. R. E. NUSBAUM, J. W. HACKETT, AND D. L. FRY, *Research Laboratories Division, General Motors Corporation.*

To analyze fabricated metal parts and various samples of stock by means of the spectrograph, a universal sampling method is needed. In order to satisfy this requirement for ferrous alloys, a process of preparing briquets from metal grindings has been developed. This process consists of reducing some of the sample to metal grindings, forming two briquets, and analyzing the latter with spark excitation. It was found that briquets formed from very small particles gave the most reproducible analyses. For the analyses made in this work, particles passing a 150-mesh screen were used. The grinding process appears to be the only way in which samples of wide degrees of hardness can be reduced to fine particles with adequate speed. Errors due to contamination of the sample by particles of grinding wheel are reduced to a minimum: first, by selecting grinding wheels which contain little or none of the elements sought in the analysis; and second, by separating nearly all of the grinding material from the sample. A magnetic method is used for making this separation on samples, such as cast iron, which can be magnetized; and an electrostatic method is used for samples, such as stainless steel, which cannot be magnetized. The time required for making two briquets from a single sample is 20 to 25 minutes. Experiments indicate that errors due to segregation are essentially eliminated by this method of analyses. Also, a series of standard samples can be obtained by diluting a sample of a known high concentration of the element sought with a sample of a known low concentration of this element. This

method has been used to analyze cast iron and stainless steel for manganese and silicon, and a modified SAE 4150 steel for manganese, silicon, molybdenum, and chromium. The standard deviations ranged from 1.4 percent to 4.7 percent.

31. The Spectrographic Flat Surface Sparking Technique of Steel Analysis. CHARLES L. GUETTEL, *Rock Island Arsenal Laboratory, Rock Island, Illinois.*

A routine testing laboratory method of steel analysis, involving the use of a spectrographic flat surface sparking technique and a commercial grating spectrograph, is described. The electrode system consists of a pointed carbon acting as a counter electrode to the flat surface of the specimen serving as the other electrode. An air blast blown into the analytical gap is used in conjunction with a high voltage controlled spark source. The surface of the specimen is prepared on a sanded polishing wheel or a surface grinder. Concentration ranges, element lines, and iron internal standard lines are given for the percentage determinations of manganese, nickel, chromium, molybdenum, silicon, vanadium, aluminum, copper, and zirconium in steel. Complete excitation and film developing conditions are presented. A discussion concerning the accuracy and the effects of certain variables such as changes in humidity, specimen size and surface, and extraneous elements, also are included. The technique has been used with satisfaction for over three years.

BIBLIOGRAPHY: F. G. Barker, J. Iron and Steel Institute **139**, 228 (1939).

32. An Application of the Spectrograph to the Inspection of Fabricated Iron and Steel. H. F. KINCAID, *International Harvester Company.*

In the modern testing laboratory the spectrographic technique must be developed to a degree whereby it may deal effectively with almost any problem that should arise. This requirement is fulfilled, quite satisfactorily, with a commercial grating spectrograph and the use of the flat surface method of sampling. This type of sampling adds to the flexibility of the spectrographic technique in that it will accommodate odd sizes and shapes of material to be inspected, and permits the non-destructive testing of finished parts. A successful commercial installation and its operation are described. Influence of length of preburn, humidity changes, voltage fluctuation, etc. on the accuracy of spectrographic analysis is discussed in detail with experimental data. The analytical procedure involves the use of a single iron reference line for the quantitative determinations of manganese, chromium, nickel, molybdenum, vanadium, copper, aluminum, and silicon in steel. The same technique is used for the quantitative determinations of silicon and manganese in gray iron foundry control work and the testing of miscellaneous castings. Tables illustrating the lines used, excitation conditions, developing procedure, and the accuracy to be expected of the quantitative determinations of five elements from a single spectrogram will be presented.

BIBLIOGRAPHY:
F. G. Barker, J. Iron and Steel Institute.
G. R. Harrison. *M. I. T. Wavelength Tables.*

33. Spectrophotometric Experiments with an Ultraviolet Multiplier Photo-Tube. R. J. PFISTER, *Mineral Industries Experiment Station, The Pennsylvania State College.*

Substitution of an RCA type C-7045 multiplier phototube (similar to the RCA 931 tube except for an ultraviolet transparent envelope) for the GE type FJ-405 sodium photo-tube in an ultraviolet recording photoelectric grating monochromator[1] did not permit recording of ultraviolet absorption curves. The difficulty was believed to be due to the greater sensitivity of the multiplier photo-tube for the short wave-length ultraviolet and the unexpected scattering of the short ultraviolet by the diffraction grating. A Gaertner type L234 quartz prism monochromator afforded one solution to this problem. The monochromator was equipped with special pulley drives which turned the prism and continuously adjusted the collimating and focusing lenses in synchronism with the prism. With the use of a recording galvanometer,[2] the transmission curves of toluene and ethyl benzene could now be obtained. Slits of approximately 0.02 mm were used for these spectra. Emission spectra taken of the commercial type H-4 high pressure mercury vapor lamp indicate that the lines 2753 and 2760A can readily be separated and the triplet 3126–31–45A can be partially resolved using a slit of about 0.005 mm.

[1] R. J. Pfister and D. H. Rank, Bull. Am. Phys. Soc. **17**, No. 3, 9 (1942).
[2] R. J. Pfister and D. H. Rank, J. Opt. Soc. Am. **32**, 397 (1942).

34. Fluorescence and Absorption Spectral Data for Pterin-like Pigments Synthesized by a Taxigenic Diphtheria Bacillus and Isolated by Chromatographic Analysis. M. O'L. CROWE AND A. WALKER, *Division of Laboratories and Research, New York State Department of Health, Albany.*

Spectroscopic and other data for certain fluorescent pigments that are synthesized by the diphtheria bacillus and isolated from a toxic culture filtrate of the diphtheria bacillus by means of ultrafiltration and extraction, and ultimately separated by means of chromatographic analysis, demonstrate the similarity of certain of these pigments to pterines isolated from the wings of butterflies and the integument of other insects.

35. Ultraviolet Absorption by the Sulfonamides and their Inhibitor, Para-aminobenzoic Acid. GLADYS A. ANSLOW AND ROSA KOHLS CONLON, *Smith College.*

The effects of increasing the alkalinity of the sulfa-drugs and of the inhibitor, para-aminobenzoic acid, and of mixing the drugs with the inhibitor on their absorption spectra have been investigated. All the sulfonamides yield bands of nearly equal intensity, ascribable to the sulfanilyl group, with maxima near 39,000 cm^{-1} in water solutions, which are practically identical in the spectra of sulfanilimide and sulfaguanadine, in which the effect of the *R* groups on the sulfanilyl must be similar, and appear at slightly lower wave numbers in sulfapyridine, sulfadiazine, and sulfathiozole, in which *R* is a heterocyclic ring. In addition, the spectra of sulfapyridine shows a 41,650 cm^{-1} band and that of sulfadiazine a 42,000 cm^{-1} band in locations near to those of pyridine and pyrimidine, respectively; and sulfathiozole

exhibits a thiozole band with center near 38,450 cm^{-1}. Increases in *p*H increase the intensity and produce unusual energy shifts. All sulfanilyl bands move progressively to higher energy locations, the pyridine and pyrimidine bands shift consistently to slightly lower energy regions, but the thiozole band shifts first to lower but with greater alkalinity to higher energy regions. The *p*-aminobenzoic acid band, on the other hand, shifts first to higher and then to intermediate energy regions. The latter shifts may result from the effect on the ring of changes in the resonance energy of the carboxylic group consequent to increased ionization at moderate alkalinity and increased salt formation at higher *p*H. The sulfonamide effects are less easily explained. Absorption by mixtures of the inhibitor and the sulfonamides is not additive. The *p*-aminobenzoic acid band appears nearly suppressed, except in alcohol solutions, but may partly follow the alkaline shift and merge with weakened sulfanilyl bands.

36. Zeeman Effects in the Arc, First Spark, and Second Spark Spectra of Yttrium. J. RAND MCNALLY, JR., AND GEORGE R. HARRISON, *Massachusetts Institute of Technology.*

Exposures to the yttrium arc in silver, burned in the Bitter electromagnet at fields of 85,000 oersteds, yielded interpretable Zeeman effects of the first three spectra of yttrium—Yt I, Yt II, Yt III. Zeeman patterns arising from transitions between energy states of the doubly-ionized atom were observed in the range 2950–2360A. Landé *g* factors were calculated for the 4*d*, 5*s*, and 5*p* energy levels. Magnetic splitting of lines in the first spark spectrum of yttrium permitted the determination of *g* values for 59 of the 61 known energy levels. Zeeman effects of the arc lines of yttrium were observed for lines of high intensity only. The inclusion of data of B. E. Moore and W. F. Meggers gave fairly complete information of the Zeeman effects of Yt I. Over-all *g* sums pertaining to the singly-ionized atom verify the *g* sum rule exceptionally well in spite of large variations of individual *g* values from their theoretical *LS* values. Term classifications were verified in the majority of cases.

BIBLIOGRAPHY:
1. B. E. Moore, Astrophys. J. **28**, 1 (1908); Ann. d. Physik **25**, 309 (1908).
2. S. Popow, Ann. d. Physik **45**, 163 (1914).
3. W. F. Meggers and B. E. Moore, J. Wash. Acad. Sci. **15**, 207 (1925).
4. I. S. Bowen and R. A. Millikan, Phys. Rev. **28**, 923 (1926).
5. W. F. Meggers and H. N. Russell, Bur. Stand. J. Research **2**, 733 (1929) (RP 55).
6. G. R. Harrison and F. Bitter, Phys. Rev. **57**, 15 (1940).
7. G. R. Harrison and J. R. McNally, Jr., Phys. Rev. **58**, 703 (1940)
8. G. R. Harrison, J. Opt. Soc. Am. **25**, 169 (1935); Rev. Sci. Inst. **9** 15 (1938); G. R. Harrison and J. P. Molnar, J. Opt. Soc. Am. **33** 43 (1940).

37. Forbidden Line λ2270 of Neutral Mercury. S. MROZOWSKI, *University of Chicago.*

The forbidden line λ2270($6^1S_0-6^3P_2$) of mercury was obtained in a water-cooled mercury arc. Very high vacuum and low density of current are necessary to get the line with relatively high intensity. The hyperfine structure of the line was investigated by means of an aluminized Perot-Fabry etalon crossed with a medium quartz spectrograph. Three components can be seen on the plates: 0 (5), ~300 (2) and $\sim 620\times10^{-3}$ cm^{-1} (1). The intensities given in parenthesis are values roughly estimated. The structure can be very well explained by the assumption that the line is emitted by the odd isotopes of mercury only; the breakdown of the *j* selection rule being caused by the extremely weak hyperfine structure perturbation occurring between the levels 6^3P_2 and 6^3P_1. The fourth component predicted on basis of such an assumption[1] is too weak to be recorded on the photographic plate. The line λ2270 is hence analogous to the previously studied forbidden line λ2967.52[2] of HgI, but its transition probability is very much smaller (about 10^6 times).

[1] F. R. Einaudi, Atti R. Accad. Lincei **17**, 552 (1933).
[2] S. Mrozowski, Phy. Rev. **61**, 735 (1942).

Author Index of Papers Presented at the Annual Meeting

Adams, Elliot Q.—No. 3
Anslow, Gladys A. and Rosa Kohls Conlon—No. 35
Babcock, Harold D.—No. 7
Ballard, James W., H. I. Oshry, and H. H. Schrenk—No. 17
Barnes, Norman F.—No. 11
—— see Sitterson, Henry A.
Boettner, E. A. and G. P. Brewington—No. 23
Brewington, G. P.—see Boettner, E. A.
Caldecourt, V. J.—see Saunderson, J. L.
Campbell, H. N.—see Hampton, Robert R.
Conlon, Rosa Kohls—see Anslow, Gladys A.
Corliss, Charles H.—see Scribner, Bourdon F.
Crowe, M. O'L. and A. Walker—No. 34
Dietert, H. W.—see Hasler, M. F.
Farnsworth, Dean, Lt., USN—No. 1
Fry, D. L.—see Nusbaum, R. E.
Fry, Glenn A.—No. 2
Guettel, Charles L.—No. 31
Hackett, J. W.—see Nusbaum, R. E.
Hampton, Robert R. and H. N. Campbell—No. 24
Harrison, George R.—see McNally, J. Rand
Hasler, M. F. and H. W. Dietert—No. 22
Hasler, M. F. and J. W. Kemp—No. 28
Herzberger, Max—No. 10
Hess, T. M. and L. G. Reinhardt—No. 19
Hodge, Edwin S.—No. 16
Hunter, Richard S.—No. 15
Kemp, J. W.—see Hasler, M. F.
Kincaid, H. F.—No. 32

Leichtle, P. A.—No. 20
McNally, J. Rand, Jr., and George R. Harrison—No. 36
Minkowski, R.—No. 8
Mohler, Orren C.—No. 9
Moon, Parry—No. 4
Mrozowski, S.—No. 37
Nusbaum, R. E., J. W. Hackett, and D. L. Fry—No. 30
Oshry, H. I.—see Ballard, James W.
Parsons, S. L.—No. 18
Pfister, R. J.—No. 33
Read, H. S.—No. 26
Reiss, Max—No. 14
Saunderson, J. L. and V. J. Caldecourt—No. 25
Schrenk, H. H.—see Ballard, James W.
Scribner, Bourdon F. and Charles H. Corliss—No. 29
Sitkin, Alexander—see Sloviter, Henry A.
Sitterson, C. B. and Norman F. Barnes—No. 12
Sloviter, Henry A. and Alexander Sitkin—No. 27
Spencer, Domina Eberle—Nos. 5 and 6
Stephens, Robert E.—No. 13
Walker, A.—see Crowe, M. O'L.
Wilson, Mabel F.—No. 21

Minutes of the Twenty-Second Meeting of the Board of Directors of the Optical Society of America, Incorporated

HOTEL WEBSTER HALL, PITTSBURGH, PENNSYLVANIA, OCTOBER 7, 1943

IN addition to the transaction of routine business, the following items of general interest to members of the Society were taken up.

Announcements by Secretary

Results of letter ballots:

1. Approval of appointment of Frank Benford (Chairman), Gladys M. Anslow, O. S. Duffendack, Selig Hecht, Harry Helson, and Elmer Hutchisson as members of the Adolph Lomb Medal Committee.
2. Approval of appointment of G. R. Harrison, E. F. Kingsbury, Edison Pettit, W. N. St. Peter, E. Ward Tillotson, and Mary E. Warga as members of the Program Committee for the Annual Meeting, under the Chairmanship of Dr. G. W. Moffitt.
3. Approval of appointment of Selig Hecht to succeed himself as O.S.A. representative on the Council of the A.A.A.S. for a term to expire in February, 1945
4. Approval of appointment of Brian O'Brien to succeed himself as O.S.A. representative on the Sectional Committee of the A.S.A. on Photography, Z-38, for a term to expire in October, 1945.
5. Approval of appointment of A. H. Pfund (Chairman), E. F. Barker, R. B. Barnes, H. H. Nielsen, and Norman Wright as members of the Program Committee for the Winter Meeting, 1944.

Appointment of Tellers for 1943 election:

E. R. Schwarz, Chairman — Donald P. Severance
Hans Mueller — G. G. Harvey, Alternate

Report of Tellers

The report indicated the election of the following officers:

For a term of two years: (beginning October 10, 1943)

President: A. H. Pfund
Vice President: G. R. Harrison

For a term of four years: (beginning October 10, 1943)

Members-at-large of the Board of Directors:

Wallace Brode — Selig Hecht

Membership

The Secretary reported that between January 1 and October 6 there had been a net gain in membership of 138. A chart of the membership growth showed that the increase during this ten-month period was at a rate that is only slightly less than has been maintained for the last three years.

On recommendation of the Committee on Transfer, 46 associate members were approved for transfer to regular membership. The names of those transferred will appear in the annual report of the Secretary.

Appointment of Associate Editors

The following Associate Editors of the Journal were appointed for a term of six years commencing January 1, 1944:

Frank Benford — A. C. Hardy
B. H. Carroll — L. A. Jones
O. S. Duffendack — W. F. Meggers
W. E. Forsythe — Brian O'Brien

ARTHUR C. HARDY, *Secretary*

Author Index to Volume 33

References with (A) are to abstracts of papers presented at meetings of the Optical Society of America; (T) designates title only; (L) designates Letters to the Editor.

Adams, Elliot Q. Chromatic valence as a correlate of Munsell chroma—683(A)

—— Letter to the editor—174(L)

Amann, Rudolf. Letter to the editor—439(L)

Ames, A., Jr. (see Ogle, Kenneth N.)—137

Anderson, William Theodore, Jr. Mercury arc lamps: effect of electrode on energy distribution—104

Anslow, Gladys A. and Rosa Kohls Conlon. Ultraviolet absorption by the sulfonamides and their inhibitor, para-aminobenzoic acid—689(A)

Babcock, Harold D. Bright diffraction gratings—683(T)

Ballard, James W., H. I. Oshry, and H. H. Schrenk. Sampling, mixing, and grinding techniques in the preparation of samples for quantitative analysis by x-ray diffraction and spectrographic methods—667, 686(A)

Barnes, Norman F. New optical bench—684(A)

—— (see Sitterson, C. B., Jr.)—684(A)

Beggs, E. W. Activating light sources for luminescent materials—61

Benford, Frank. Illuminagraphic camera—440

—— Magnification and brightness of images on the retina—245(L)

Bennett, A. H. Development of the microscope objective—123; Errata—543

Bennett, H. F. Computation of polynomial functions by summation of finite differences—519

Boettner, E. A. and G. P. Brewington. Application of multiplier photo-tubes to quantitative spectrochemical analysis—687(A)

Brewington, G. P. (see Boettner, E. A.)—687(A)

Caldecourt, V. J. (see Saunderson, J. L.)—687(A)

Campbell, H. N. (see Hampton, Robert)—687(A)

Committee on Colorimetry. Concept of color—544

Conlon, Rosa Kohls (see Anslow, Gladys A.)—689(A)

Corliss, Charles H. (see Scribner, Bourdon F.)—515, 688(A)

Creech, Hugh J. (see Jones, R. Norman)—209

Crosswhite, H. M. (see Dieke, G. H.)—425

Crowe, M. O'L. and A. Walker. Fluorescence and absorption spectral data for pterin-like pigments synthesized by a toxigenic diphtheria bacillus and isolated by chromatographic analysis—689(A)

Dieke, G. H. and H. M. Crosswhite. Use of iron lines as intensity standards—425

Dietert, H. W. (see Hasler, M. F.)—218, 687(A)

—— **and M. F. Hasler.** Electronic voltage regulator for precise densitometric measurements—45

Dimmick, Forrest Lee. Methodology in test preparation—308, 350(T)

Dorst, Paul W. Halftone densities—436

Duntley, Seibert Q. Mathematics of turbid media—252

Ellis, James H. Note on ray tracing by machine—665

Evans, Ralph M. Visual processes and color photography—579

Farnsworth, Dean. Description of a subject congenitally deficient in violet-yellow vision—350(A)

—— Farnsworth-Munsell 100-hue and dichotomous tests for color vision—350(T), 568

—— Subjective boundaries of the spectral colors—682(A)

Foss, Carl E. (see Granville, Walter C.)—376

Francis, B. G. Necrology: Carl Pfanstiehl—422

Frisch, S. Optical physics in the U.S.S.R.—637(L)

Fry, D. L. (see Nusbaum, R. E.)—688(A)

Fry, Glenn A. Quantitative formulations of color mixture and wave-length discrimination data for dichromats—682(A)

Gardner, I. C. Necrology: Alfred Nelson Finn—424

Gibb, Thomas R. P., Jr. Book review—639

Gibson, Kasson S. Necrology: Harry John McNicholas—424

—— (see Kelly, Kenneth L.)—355

Godlove, I. H. Analysis of some of the chemists' concepts of color—351(A)

—— Conditions for dyestuffs spectrophotometry and some techniques—351(A)

Granville, Walter C., Dorothy Nickerson, and Carl E. Foss. Trichromatic specifications for intermediate and special colors of the Munsell system—376

Grimm, H. H. (see Rank, D. H.)—31

Grossman, H. H., R. A. Sawyer, and H. B. Vincent. Spectrochemical light source errors and their compensation—185

Guettel, Charles L. Spectrographic flat surface sparking technique of steel analysis—689(A)

Hackett, J. W. (see Nusbaum, R. E.)—688(A)

Hall, A. G. Book review—269

Hampton, Robert and H. N. Campbell. Measurement of spectral intensities in the direct-current arc—687(A)

Hardy, Arthur C. Annual report of the secretary of the Optical Society of America, for the calendar year 1942—340

—— Flux calculations in optical systems—71

—— Minutes of the twenty-first meeting of the Board of Directors of the Optical Society of America, Incorporated—335

—— Minutes of the twenty-second meeting of the Board of Directors of the Optical Society of America, Incorporated—691

—— Optical Society of America, Incorporated. Notice of election of officers—244

Hardy, LeGrand H. ISCC single judgment test for red-green discrimination—350(T), 512

Harrison, George R. Report of the editor of publications—336

—— (see McNally, J. Rand, Jr.)—690(A)

Harrison, V. G. W. Comment on "Meetings of scientific societies in wartime"—245(L)

Hasler, M. F. (see Dietert, H. W.)—45

—— **and H. W. Dietert.** Direct reading instrument for spectrochemical analysis—687(A)
—— **and H. W. Dietert.** New spectroscopic source unit—218
Hasler, M. F. and J. W. Kemp. Spectrochemical analysis of metals with the multi-source unit—688(A)
Hecht, Selig. Book review—353
——, **Simon Shlaer, and James C. Peskin.** Red-green and yellow equation for normal and color-blind observers—350(T)
Helson, Harry. Some factors and implications of color constancy—348(T), 555
Herzberger, Max. Gaussian optics and Gaussian brackets—651, 684(A)
Hess, T. M. and L. G. Reinhardt. Spectrographic analysis of magnesium alloys—686(A)
Hodge, Edwin S. Compendium of line pairs and operating conditions used in spectrographic analysis—656, 685(A)
Hopfield, John J. Light-and-shadow box as a visual aid in measuring spectra—113
Hughes, Ray C. Preparation of purified inorganic compounds for use in spectrographic standards—49
Hulbert, E. O. Letter to the editor—505(L)
—— Propagation of radiation in a scattering and absorbing medium—42
Hunter, Richard S. Geometric identification of reflection and transmission measurements—685(A)
Ingle, George W. Supplementary selected ordinate method of tristimulus integration—349(A)
Ives, Herbert E. Impact of a wave-packet and a reflecting particle—163
James, T. H. (see Kornfeld, G.)—615
Jaycox, E. K. (see Ruehle, A. E.)—109
Jobe, Frederick W. Method of testing color vision using colored transparencies and standard conditions of observation—350(T)
Jones, Loyd A. Historical background and evolution of the colorimetry report—534
Jones, R. Norman and Hugh J. Creech. Ultraviolet absorption spectra of protein solutions—209
Judd, Deane B. Facts of color-blindness—294, 350(T)
—— Standard response functions for protanopic and deuteranopic vision—351(A)
—— (see Newhall, Sidney M.)—349(A), 385
Kelly, Kenneth L. Color designations for lights—349(A), 627
——, **Kasson S. Gibson, and Dorothy Nickerson.** Tristimulus specification of the *Munsell Book of Color* from spectrophotometric measurements—355
Kemp, J. W. (see Hasler, M. F.)—688(A)
Kincaid, H. F. Application of the spectrograph to the inspection of fabricated iron and steel—689(A)
Kornfeld, G. and T. H. James. Herschel effect in relation to a critical size of development centers—615
Kueck, Paul D. (see Lorenz, Egon)—663
Leichtle, P. A. Spectrographic determination of trace elements in cartridge brass and admiralty brass—686(A)
Leighton, Philip A. and David H. Volman. Absorption tube for the investigation of gases at low pressures—79
Levy, Saul. Note on the correlation method in quantitative spectrum analysis—637(L)
Lowry, Earl M. Effect of hue on dark adaptation—619
Lorenz, Egon and Paul D. Kueck. Hydrogen discharge tube with metal capillary—663
MacAdam, D. L. Graphical representation of small color differences—675
—— Hue discrimination test for anomalous color vision—350(T)
—— Specification of small chromaticity differences—18
Mahan, Archie I. Mathematical proof of Stokes' reversibility principle—621
May, Albert. Solarization with x-rays and visible light—81
McNally, J. Rand, Jr., and George R. Harrison. Zeeman effects in the arc, first spark, and second spark spectra of yttrium—690(A)
McRae, D. B. Measurement of transmission and contrast in optical instruments—229
Minkowski, R. Schmidt systems as spectrograph cameras—683(T)
Mohler, Orren C. McGregor spectrograph of the McMath-Hulbert Observatory—683(T)
Monk, George S. Necrology: Hans Georg Beutler—423
Moon, Parry. Analytic approximation of standard trichromatic data—351(A)
—— New formulation of color harmony—683(A)
—— New methods of calculating illumination—115
—— **and Domina Eberle Spencer.** Analytical representation of standard response curves—89
—— **and Domina Eberle Spencer.** Metric based on the composite color stimulus—270
—— **and Domina Eberle Spencer.** Metric for colorspace—260
—— **and Domina Eberle Spencer.** Specification of foveal adaptation—444
Mrozowski, S. Forbidden line λ2270 of neutral mercury—690(A)
Murray, Elsie. Evolution of color vision tests—316, 350(T)
Neuhaus, C. J. Study of the applicability of the pressed pellet electrode method to the spectrographic analysis of nickel alloys—167
Newhall, Sidney M. (see Nickerson, Dorothy)—349(A), 419
——, **Dorothy Nickerson, and Deane B. Judd.** Final report of the O.S.A. subcommittee on the spacing of the Munsell colors—349(A), 385
Nickerson, Dorothy. Symposium on color-blindness: Foreword—293
—— (see Granville, Walter C.)—376
—— (see Kelly, Kenneth L.)—355
—— (see Newhall, Sidney M.)—349(A), 385
—— **and Sidney M. Newhall.** Psychological color solid—349(A), 419
Nicoll, F. H. and F. E. Williams. Properties of low reflection films produced by the action of hydrofluoric acid vapor—434
Nusbaum, R. E., J. W. Hackett, and D. L. Fry. Use of briquets formed from metal grindings for the spectrographic analysis of steel—688(A)

Ogle, Kenneth N. Some aspects of the eye as an image-forming mechanism—348(T), 506
—— **and A. Ames, Jr.** Opthalmic lens testing instrument—137
Oshry, H. I. (see Ballard, James W.)—667, 686(A)
Osterberg, Harold. Conditions for high contrast in the Twyman-Green interferometer—348(A)
Parsons, S. L. Spectrographic determination of thorium in tungsten filament wire—659, 686(A)
Peskin, James C. (see Hecht, Selig)—350(T)
Pfister, R. J. Spectrophotometric experiments with an ultraviolet multiplier photo-tube—689(A)
—— (see Rank, D. H.)—31
Platt, John R. Note on the theoretical color-blindness frequencies—679(L)
Rand, Gertrude K. Necrology: Clarence Errol Ferree—423
Rank, D. H., R. J. Pfister, and H. H. Grimm. Method for determining depolarization factors in Raman spectra—31
Read, H. S. Preliminary curve for a spectrographic plate calibration curve—687(A)
Reinhardt, L. G. (see Hess, T. M.)—686(A)
Reiss, Max. Design of a telephoto-magnifier—641, 684(A)
Royer, G. L. Necrology: Orrin W. Pineo—424
Ruehle, A. E. and E. K. Jaycox. Some observations on the use of powders in spectrochemical analysis—109
Saunderson, J. L. and V. J. Caldecourt. Accessory equipment for spectrochemical analysis—687(A)
Sawyer, R. A. (see Grossman, H. H.)—185
—— **and H. B. Vincent.** Plate calibration problems—247
Schrenk, H. H. (see Ballard, James W.)—667, 686(A)
Scott, G. C. Necrology: Robert E. Oltman—422
Scribner, Bourdon F. and Charles H. Corliss. Accuracy in the spectrographic analysis of steel—688(A)
—— **and Charles H. Corliss.** Improved electrode holder for spectrographic analysis—515
Sheard, Charles. Dark adaptation: some physical, physiological, and clinical considerations—348(T)
Shlaer, Simon (see Hecht, Selig)—350(T)
Silberstein, Ludwik. Dependence of intrinsic sensitivity on grain size—175
—— Investigations on the intrinsic properties of the color domain. II—1
—— On size-frequency distribution in photographic emulsions—35
—— Simplified formulae for scattered and rescattered sunlight—526
Sitkin, Alexander (see Sloviter, Henry A.)—687(A)
Sitterson, C. B., Jr., and Norman F. Barnes. Optical sighting gauge for the Garand rifles—684(A)
Sloviter, Henry A. and Alexander Sitkin. Improved spectrochemical method using spark excitaton—687(A)
Spencer, Domina Eberle. Adaptation in color space—10
—— Aesthetic measure in color harmony—683(A)
—— Generalized theory of color harmony—683(A)
—— Inclusion of visual adaptation in a metric for color space—349(A)
—— (see Moon, Parry)—89, 260, 270, 444
Stearns, E. I. Applications of near infra-red spectrophotometry—27
Stempel, W. M. Empirical approach to lens designs. The Huygens eyepiece—278
Stephens, Robert E. Displacement of the focal point of a lens system resulting from change of temperature—684(A)
Stevens, Paul W. Analysis of errors in a reflecting prism—74
Sweet, Monroe H. Improved procedure for the contact printing method of measuring photographic density—143
—— Spectral characteristics of optical wedges—194
Synge, J. L. Theory of the Schmidt telescope—129
Van den Akker, J. A. Wave-length calibration of spectrophotometers—257
Vincent, H. B. (see Grossman, H. H.)—185
—— (see Sawyer, R. A.)—247
Volman, David H. (see Leighton, Philip A.)—79
Walker, A. (see Crowe, M. O'L.)—689(A)
Walls, Gordon L. Factors in human visual resolution—348(T), 487
Welsh, John H. Book review—578
Williams, F. E. (see Nicoll, F. H.)—434
Wilson, Mabel F. Spectrographic determination of small amounts of impurities in magnesium chloride—686(A)
Wright, W. D. Graphical representation of small color differences—632
Zigler, Michael J. Book review—352

Analytic Subject Index to Volume 33

References with (A) are to abstracts of papers presented at meetings of the Optical Society of America; (L) designates Letters to the Editor.

Absorption, including absorption spectra
Absorption tube for the investigation of gases at low pressures, Philip A. Leighton and David H. Volman—79
Propagation of radiation in a scattering and absorbing medium, E. O. Hulbert—42
Ultraviolet absorption spectra of protein solutions, R. Norman Jones and Hugh J. Creech—209
Ultraviolet absorption by the sulfonamides and their inhibitor, para-aminobenzoic acid, Gladys A. Anslow and Rosa Kohls Conlon—689(A)
Announcements
New headquarters building of the American Institute of Physics—626
New house organ of interest to spectrographers—620
New institute for fundamental research in glass—614
New precision instrument for measuring gloss—554
Book reviews
Polarimetry, Saccharimetry and the Sugars by Frederick J. Bates and Associates of the National Bureau of Standards, T. R. P. Gibb, Jr.—639
Prism and Lens Making by F. Twyman, A. G. Hall—269
Sensation and Perception in the History of Experimental Psychology by E. G. Boring, Michael J. Zigler—352
Vertebrate Eye and its Adaptive Radiation by Gordon Lynn Walls, Selig Hecht—353
Vertebrate Photoreceptors by Samuel R. Detwiler, John H. Welsh—578

Color and colorimetry
Adaptation in color space, Domina Eberle Spencer—10
Aesthetic measure in color harmony, Domina Eberle Spencer—683(A)
Analysis of some of the chemists' concepts of color, I. H. Godlove—351(A)
Analytic approximation of standard trichromatic data, Parry Moon—351(A)
Analytical representation of standard response curves, Parry Moon and Domina Eberle Spencer—89
Chromatic valence as a correlate of Munsell chroma, Elliot Q. Adams—683(A)
Color designations for lights, Kenneth L. Kelly—349(A), 627
Concept of color, Committee on Colorimetry—544
Conditions for dyestuffs spectrophotometry and some techniques, I. H. Godlove—351(A)
Description of a subject congenitally deficient in violet-yellow vision, Dean Farnsworth—350(A)
Effect of hue on dark adaptation, Earl M. Lowry—619
Evolution of color vision tests, Elsie Murray—316
Facts of color-blindness, Deane B. Judd—294
Farnsworth-Munsell 100-hue and dichotomous tests for color vision, Dean Farnsworth—568
Final report of the OSA subcommittee on the spacing of the Munsell colors, Sidney M. Newhall, Dorothy Nickerson, and Deane B. Judd—349(A), 385
Generalized theory of color harmony, Domina Eberle Spencer—683(A)
Graphical representation of small color differences, D. L. MacAdam—675
Graphical representation of small color differences, W. D. Wright—632
Historical background and evolution of the colorimetry report, Loyd A. Jones—534
Inclusion of visual adaptation in a metric for color space, Domina Eberle Spencer—349(A)
Investigations on the intrinsic properties of the color domain. II, L. Silberstein—1
Methodology in test preparation, Forrest Lee Dimmick—308
Metric based on the composite color stimulus, Parry Moon and Domina Eberle Spencer—270
Metric for colorspace, Parry Moon and Domina Eberle Spencer—260
New formulation of color harmony, Parry Moon—683(A)
Note on theoretical color-blindness frequencies, John R. Platt 679(L)
Psychological color solid, Dorothy Nickerson and Sidney M. Newhall—349(A), 419
Quantitative formulations of color mixture and wavelength discrimination data for dichromats, Glenn A. Fry—682(A)
Single judgment test for red-green discrimination, LeGrand H. Hardy—512
Some factors and implications of color constancy, Harry Helson—555
Specification of small chromaticity differences, David L. MacAdam—18
Subjective boundaries of the spectral colors, Dean Farnsworth—682(A)
Supplementary selected ordinate method of tristimulus integration, George W. Ingle—349(A)
Symposium on color-blindness. Foreword, Dorothy Nickerson—293
Trichromatic specifications for intermediate and special colors of the Munsell system, Walter C. Granville, Dorothy Nickerson, and Carl E. Foss—376
Tristimulus specification of the *Munsell Book of Color* from spectrophotometric measurements, Kenneth L. Kelly, Kasson S. Gibson, and Dorothy Nickerson—355
Visual processes and color photography, Ralph M. Evans—579

Editorial comment
Report of the colorimetry committee—533

Erratum
Development of the microscope objective, A. H. Bennett—543

Eye
Description of a subject congenitally deficient in violet-yellow vision, Dean Farnsworth—350(A)
Factors in human visual resolution, Gordon L. Walls—487
Magnification and brightness of images on the retina, Frank Benford—245(L)
Note on theoretical color-blindness frequencies, John R. Platt—679(L)
Some aspects of the eye as an image-forming mechanism, Kenneth N. Ogle—506
Specification of foveal adaptation, Parry Moon and Domina Eberle Spencer—444
Standard response functions for protanopic and deuteranopic vision, Deane B. Judd—351(A)

Fluorescence and phosphorescence
Activating light sources for luminescent materials, E. W. Beggs—61
Fluorescence and absorption spectral data for pterin-like pigments synthesized by a toxigenic diphtheria bacillus and isolated by chromatographic analysis, M. O'L. Crowe and A. Walker—689(A)

Illumination
Color designations for lights, Kenneth L. Kelly—349(A), 627
New methods of calculating illumination, Parry Moon—115

Interferometry
Conditions for high contrast in the Twyman-Green interferometer, Harold Osterberg—348(A)

Lenses
Design of a telephoto-magnifier, Max Reiss—641, 686(A)
Development of the microscope objective, A. H. Bennett—123
Displacement of the focal point of a lens system resulting from change of temperature, Robert E. Stephens—684(A)

Lenses (*continued*)
Empirical approach to lens designs. The Huygens eyepiece, W. M. Stempel—278
Ophthalmic lens testing instrument, Kenneth N. Ogle and A. Ames, Jr.—137
Theory of the Schmidt telescope, J. L. Synge—129
Letters to the editor
Comment on Letter to the Editor by Frank Benford, J. Opt. Soc. Am. **33**, 245 (1943), Rudolf Amann—439(L)
Comment on "Meetings of scientific societies in wartime," V. G. W. Harrison—245(L)
Comment on "Mercury arc lamps: effect of energy distribution" by W. T. Anderson, Jr., J. Opt. Soc. Am. **33**, 104 (1943), Elliot Q. Adams—174(L)
Correction "Propagation of radiation in a scattering and absorbing medium," J. Opt. Soc. Am. **33**, 42–45 (1943), E. O. Hulburt—505(L)
Magnification and brightness of images on the retina, Frank Benford—245(L)
Note on the correlation method in quantitative spectrum analysis, Saul Levy—637(L)
Note on theoretical color-blindness frequencies, John R. Platt—679(L)
Optical physics in the U.S.S.R., S. Frisch—637(L)
Light sources, properties and measurements
Activating light sources for luminescent materials, E. W. Beggs—61
Color designations for lights, Kenneth L. Kelly—349(A), 627
Comment on "Mercury arc lamps: effect of energy distribution" by W. T. Anderson, Jr., J. Opt. Soc. Am. **33**, 104 (1943), Elliot Q. Adams—174(L)
Hydrogen discharge tube with metal capillary, E. Lorenz and P. S. Kueck—663
Measurement of spectral intensities in the direct-current arc, Robert R. Hampton and H. N. Campbell—687(A)
Mercury arc lamps: effect of electrode on energy distribution, William Theodore Anderson, Jr.—104
New spectroscopic source unit, M. F. Hasler and H. W. Dietert—218
Microscopy
Development of the microscope objective, A. H. Bennett—123
Necrology
Anthes, E. H.—423
Beutler, Hans Georg—423
Ferree, Clarence Errol—423
Finn, Alfred Nelson—424
Frederick, Charles W.—422
McNicholas, Harry John—424
Oltman, Robert E.—422
Pfansteihl, Carl—422
Pineo, Orrin W.—424
Optical Society of America
Abstracts of contributed papers presented before sessions of the Optical Society of America—348, 682
Annual report of the editor of publications—336
Annual report of the secretary of the O. S. A., Inc., for 1942—340
Annual report of the treasurer of the O. S. A., Inc., for 1942—337
Minutes of the twenty-first meeting of the Board of Directors of the Optical Society of America, New York, New York, March 4, 1943—335
Minutes of the twenty-second meeting of the Board of Directors of the Optical Society of America, Pittsburgh, Pennsylvania, October 7, 1943—691
Notice of election of officers—244
Proceedings of the twenty-eighth annual meeting of the Optical Society of America, Philadelphia, Pennsylvania, October 7–9, 1943—680
Proceedings of the winter meeting of the Optical Society of America, New York, New York, March 5–6, 1943—344
Optical testing
Displacement of the focal point of a lens system resulting from change of temperature, Robert E. Stephens—684(A)
Measurement of transmission and contrast in optical instruments, D. B. McRae—229
New optical bench, Norman F. Barnes—684(A)
Ophthalmic lens testing instrument, Kenneth N. Ogle and A. Ames, Jr.—137
Optical sighting gauge for the Garand rifle, C. B. Sitterson, Jr., and Norman F. Barnes—684(A)
Optics, general
Comment on "Meetings of scientific societies in wartime," V. G. W. Harrison—245(L)
Mathematical proof of Stokes' reversibility principle, A. I. Mahan—621
Optical physics in the U.S.S.R., S. Frisch—637(L)
Optics, geometrical
Analysis of errors in a reflecting prism, Paul W. Stevens—74
Comment on Letter to the Editor by Frank Benford, J. Opt. Soc. Am. **33**, 245 (1943), Rudolf Amann—439(L)
Computation of polynomial functions by summation of finite differences, H. F. Bennett—519
Design of a telephoto-magnifier, Max Reiss—641, 684(A)
Displacement of the focal point of a lens system resulting from change of temperature, Robert E. Stephens—684(A)
Empirical approach to lens designs. The Huygens eyepiece, W. M. Stempel—278
Flux calculations in optical systems, Arthur C. Hardy—71
Gaussian optics and Gaussian brackets, Max Herzberger—651, 684(A)
Note on ray-tracing by machine, James H. Ellis—665
Theory of the Schmidt telescope, J. L. Synge—129
Optics, physical
Correction "Propagation of radiation in a scattering and absorbing medium," J. Opt. Soc. Am. **33**, 42–45 (1943), E. O. Hulburt—505(L)
Forbidden line λ2270 of neutral mercury, S. Mrozowski—690(A)
Impact of a wave-packet and a reflecting particle, Herbert E. Ives—163

Mathematics of turbid media, Seibert Q. Duntley—252
Mercury arc lamps: Effect of electrode on energy distribution, William Theodore Anderson, Jr.—104
Method for determining depolarization factors in Raman spectra, D. H. Rank, R. J. Pfister, and H. H. Grimm —31
Zeeman effects in the arc, first spark, and second spark spectra of yttrium, J. Rand McNally, Jr., and George R. Harrison—690(A)

Photography, including cameras

Dependence of intrinsic sensitivity on grain size, Ludwik Silberstein—175
Halftone densities, Paul W. Dorst—436
Herschel effect in relation to a critical size of development centers, G. Kornfeld and T. H. James—615
Illuminagraphic camera, Frank Benford—440
Improved procedure for the contact printing method of measuring photographic density, Monroe H. Sweet—143
Size-frequency distribution in photographic emulsions, Ludwik Silberstein—35
Solarization with x-rays and visible light, Albert May —81
Visual processes and color photography, Ralph M. Evans—579.

Raman effect

Method for determining depolarization factors in Raman spectra, D. H. Rank, R. J. Pfister, and H. H. Grimm —31

Reflection

Analysis of errors in a reflecting prism, Paul W. Stevens —74
Geometric identification of reflection and transmission measurements, Richard S. Hunter—685(A)
Properties of low reflection films produced by the action of hydrofluoric acid vapor, F. H. Nicoll and F. E. Williams—434

Scattering of light

Mathematics of turbid media, Seibert Q. Duntley—252
Propagation of radiation in a scattering and absorbing medium, E. O. Hulbert—42
Simplified formulae for scattered and rescattered sunlight, Ludwik Silberstein—526

Spectrographic analysis

Absorption tube for the investigation of gases at low pressures, Philip A. Leighton and David H. Volman—79
Accessory equipment for spectrochemical analysis, J. L. Saunderson and V. J. Caldecourt—687(A)
Accuracy in the spectrographic analysis of steel, Bourdon F. Scribner and Charles H. Corliss—688(A)
Application of multiplier photo-tubes to quantitative spectrochemical analysis, E. A. Boettner and G. P. Brewington—687(A)
Application of the spectrograph to the inspection of fabricated iron and steel, H. F. Kincaid—689(A)
Compendium of line pairs and operating conditions used in spectrographic analysis, Edwin S. Hodge—656, 685(A)
Direct reading instrument for spectrochemical analysis, M. F. Hasler and H. W. Dietert—687(A)
Electronic voltage regulator for precise densitometric measurements, H. W. Dietert and M. F. Hasler—45
Improved electrode holder for spectrographic analysis, Bourdon F. Scribner and Charles H. Corliss—515
Improved spectrochemical method using spark excitation, Henry A. Sloviter and Alexander Sitkin—687(A)
Light-and-shadow box as a visual aid in measuring spectra, John J. Hopfield—113
Measurement of spectral intensities in the direct-current arc, Robert R. Hampton and H. N. Campbell—687(A)
New optical bench, Norman F. Barnes—684(A)
New spectroscopic source unit, M. F. Hasler and H. W. Dietert—218
Note on the correlation method in quantitative spectrum analysis, Saul Levy—637(L)
Plate calibration problems, R. A. Sawyer and H. B. Vincent—247
Preliminary curve for a spectrographic plate calibration curve, H. S. Read—687(A)
Preparation of purified inorganic compounds for use in spectrographic standards, Ray C. Hughes—49
Sampling, mixing, and grinding techniques in the preparation of samples for quantitative analysis by x-ray diffraction and spectrographic methods, James W. Ballard, H. I. Oshry, and H. H. Schrenk—667, 686(A)
Some observations on the use of powders in spectrochemical analysis, A. E. Ruehle and E. K. Jaycox—109
Spectrochemical analysis of metals with the multisource unit, M. F. Hasler and J. W. Kemp—688(A)
Spectrochemical light source errors and their compensation, H. H. Grossman, R. A. Sawyer, and H. B. Vincent—185
Spectrographic analysis of magnesium alloys, T. M. Hess and L. G. Reinhardt—686(A)
Spectrographic determination of small amounts of impurities in magnesium chloride, Mabel F. Wilson—686(A)
Spectrographic determination of thorium in tungsten filament wire, S. L. Parsons—659, 686(A)
Spectrographic determination of trace elements in cartridge brass and admiralty brass, P. A. Leichtle—686(A)
Spectrographic flat surface sparking technique of steel analysis, Charles L. Guettel—689(A)
Study of the applicability of the pressed pellet electrode method to the spectrographic analysis of nickel alloys, C. J. Neuhaus—167
Ultraviolet absorption by the sulfonamides and their inhibitor, para-aminobenzoic acid, Gladys A. Anslow and Rosa Kohls Conlon—689(A)
Ultraviolet absorption spectra of protein solutions, R. Norman Jones and Hugh J. Creech—209
Use of briquets formed from metal grindings for the spectrographic analysis of steel, R. E. Nusbaum, J. W. Hackett, and D. L. Fry—688(A)
Use of iron lines as intensity standards, G. H. Dieke and H. M. Crosswhite—425

Spectroscopy; spectrometry; spectrophotometry
Absorption tube for the investigation of gases at low pressures, Philip A. Leighton and David H. Volman—79
Applications of near infra-red spectrophotometry, E. I. Stearns—27
Compendium of line pairs and operating conditions used in spectrographic analysis, Edwin S. Hodge—656, 685(A)
Conditions for dyestuffs spectrophotometry and some techniques, I. H. Godlove—351(A)
Electronic voltage regulator for precise densitometric measurements, H. W. Dietert and M. F. Hasler—45
Forbidden line λ2270 of neutral mercury, S. Mrozowski—690(A)
Hydrogen discharge tube with metal capillary, E. Lorenz and P. S. Kueck—663
Light-and-shadow box as a visual aid in measuring spectra, John J. Hopfield—113
New optical bench, Norman F. Barnes—684(A)
Plate calibration problems, R. A. Sawyer and H. B. Vincent—247
Spectrophotometric experiments with an ultraviolet multiplier photo-tube, R. J. Pfister—689(A)
Ultraviolet absorption by the sulfonamides and their inhibitor, para-aminobenzoic acid, Gladys A. Anslow and Rosa Kohls Conlon—689(A)
Ultraviolet absorption spectra of protein solutions, R. Norman Jones and Hugh J. Creech—209
Use of iron lines as intensity standards, G. H. Dieke and H. M. Crosswhite—425
Wave-length calibration of spectrophotometers, J. A. Van den Akker—257

Standards for photography, American
American recommended practice for dimensions of photographic paper (Roll leaders and limit signals)—Z38.1.4-1942—179
American standard for cameras (tripod connections for American cameras)—Z38.4.1-1942—180
American standard for cameras (tripod connections for European cameras with adapter for American tripods)—Z38.4.2-1942—181
American standard dimensions for amateur roll film and backing paper—Nos. 1–9, Z38.1.7-1943; Z38.1.15-1943—459
American standard dimensions for amateur roll film spools—Nos. 1–9, Z38.1.16-1943; Z38.1.24-1943—468
American standard dimensions of photographic papers (centimeter size sheets and rolls)—Z38.1.6-1943—458
American standard dimensions of photographic papers (inch-width rolls)—Z38.1.5-1943—457
American standard for distance scales marked in feet—Z38.4.3-1942—182
American standard for focal lengths of lenses: marking—Z38.4.4-1942—182
American standard method for determining photographic speed and speed number—479
American standard for shutter cable release tip and socket with straight thread—Z38.4.6-1942—184
American standard for shutter cable release tip and socket with taper thread—Z38.4.5-1942—183
Appendix (to Z38.1.7—Z38.1.24). A1. Definition of terms; A2. Designations indicating size of American-made amateur roll films—477

Transmission
Geometric identification of reflection and transmission measurements, Richard S. Hunter—685(A)
Measurement of transmission and contrast in optical instruments, D. B. McRae—229
Propagation of radiation in a scattering and absorbing medium, E. O. Hulbert—42
Spectral characteristics of optical wedges, Monroe H. Sweet—194

Vision; visibility, visual phenomena
Analytical representation of standard response curves, Parry Moon and Domina Eberle Spencer—89
Description of a subject congenitally deficient in violet-yellow vision, Dean Farnsworth—350(A)
Effect of hue on dark adaptation, Earl M. Lowry—619
Evolution of color vision tests, Elsie Murray—316
Factors in human visual resolution, Gordon L. Walls—487
Facts of color-blindness, Deane B. Judd—294
Inclusion of visual adaptation in a metric for color space, Domina Eberle Spencer—349(A)
Magnification and brightness of images on the retina, Frank Benford—245(L)
Methodology in test preparation, Forrest Lee Dimmick—308
Note on theoretical color-blindness frequencies, John R. Platt—679(L)
Some aspects of the eye as an image-forming mechanism, Kenneth N. Ogle—506
Specification of foveal adaptation, Parry Moon and Domina Eberle Spencer—444
Standard response functions for protanopic and deuteranopic vision, Deane B. Judd—351(A)
Symposium on color-blindness, Foreword, Dorothy Nickerson—293
Visual processes and color photography, Ralph M. Evans—579